Why greybody factors remain insensitive to metric perturbations despite the spectral instability of Regge poles?

即便存在谱不稳定性,为什么灰度因子对度规扰动不敏感?

Nov. 14th 2025

Virtual Infinity Seminar

Wei-Liang Qian 錢衛良 (EEL - USP)

Primarily in collaboration with

G.-R. Li (UNESP), R. Daghigh (MSU),

M. Green (MSU), J. Morey (UMN),

Q.-Y. Pan (HNNU) & R.-H. Yue (YZU)

Primarily based on arXiv: 2504.13265

An interplay between a few concepts related to QNM

- Spectral instability of black hole quasinormal modes
- Asymptotic modes, instability in the high overtones and low-lying modes
- Gravitational wave echoes, reflectionless modes, and Regge poles
- Stable observables in the context of spectral instability

Outline

- Introduction: Asymptotic black hole quasinormal modes (QNM) and spectral instability

- Greybody factor, scattering amplitude
- Regge poles and scattering process
- Implementation of the matrix method in hyperboloidal coordinates
- Greybody factor as a stable observable in terms of Regge poles

- Speculations about potential observational implications

QNMs

- QNMs are characteristic modes of a dissipative system
- They are typically manifested as one perturbs around a static or stationary state

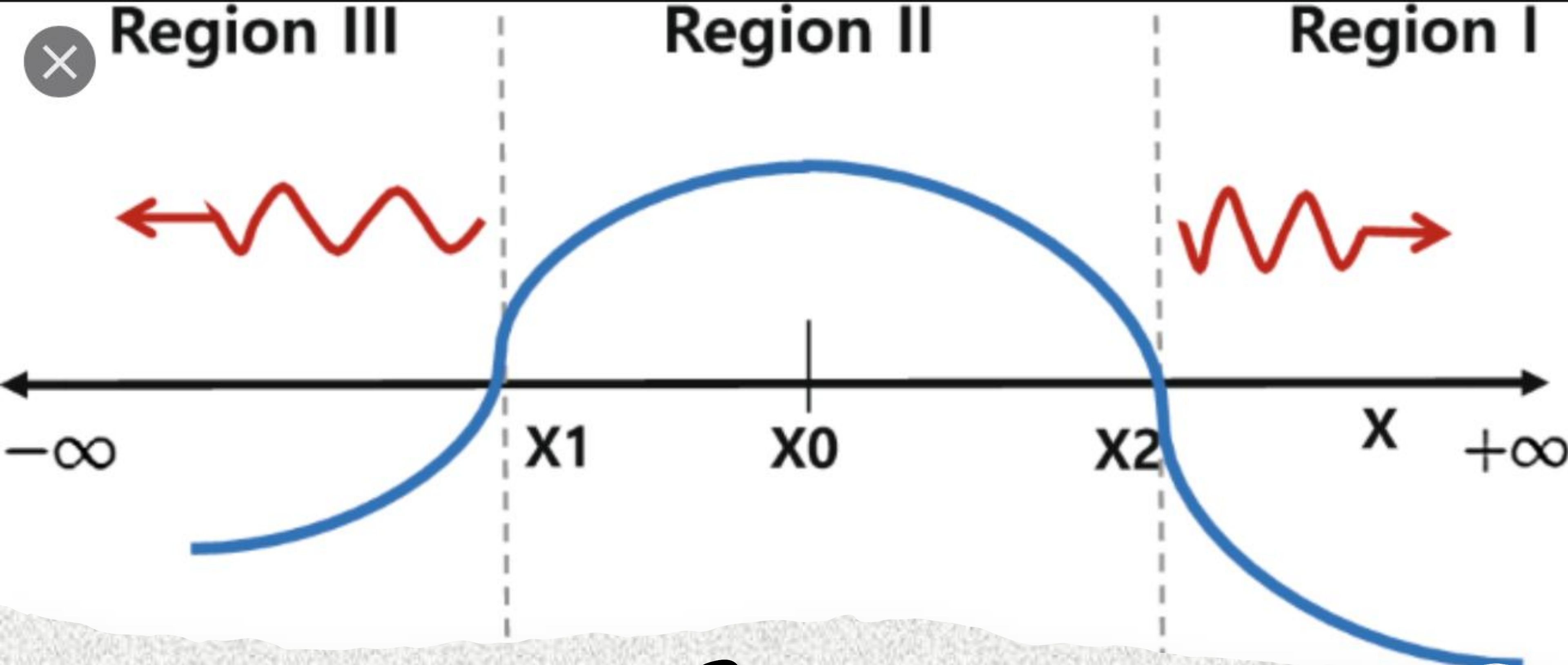
- Originally associated with stability analysis
- Classical concept
- Quantum implications due to AdS/CFT (via the Kubo formula)
- Implication of the methods used to solve for the QNM
- Structural instability

The (simplified) master equation

$$\frac{d^2\Psi_s}{dr_*^2} + \left(\omega^2 - V_s\right)\Psi_s = 0\,.$$

The Regge-Wheeler Potential

$$V_s = f\left[\frac{l(l+1)}{r^2} + (1-s^2)\left(\frac{2M}{r^3} + \frac{4-s^2}{2L^2}\right)\right]$$



The BIG picture (a potential barrier instead of a well)

Methods for QNM

(An incomplete list of primarily analytic and semi-analytic means)

- WKB
- Green function (Fourier or Laplace transforms)
- Poschl–Teller
- Singularity in transition amplitudes
- Continued fraction (expansion and convergence)
- Monodromy (Motl’s approach)
- Matrix method and pseudo spectrum method
- Time Evolution + Prony method
- Many other options

Methods for asymptotic QNM

(An incomplete list of primarily analytic and semi-analytic means)

- WKB (revised, employing Stokes lines)
- Green function (asymptotic poles)
- Poschl–Teller (entirely analytic)
- Singularity in transition amplitudes
- Continued fraction (Nollert’s refinement)
- Monodromy (Motl’s approach)
- (not so many) other options

Black hole structural instability

PHYSICAL REVIEW D VOLUME 53, NUMBER 8 15 APRIL 1996

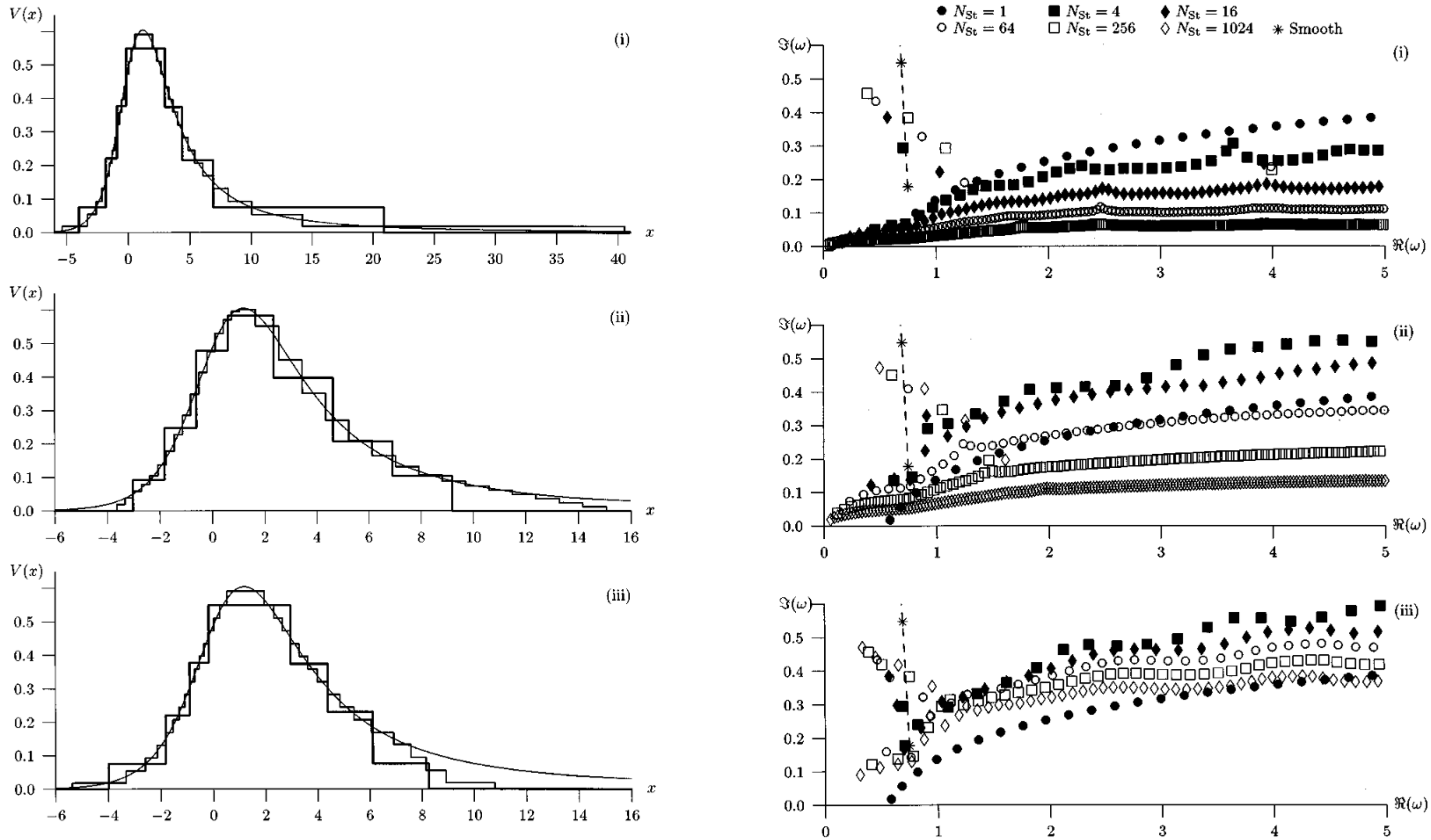
About the significance of quasinormal modes of black holes

Hans-Peter Nollert
Theoretische Astrophysik, Computational Physics, Universität Tübingen 72076 Tübingen, Germany
(Received 23 October 1995)

Quasinormal modes have played a prominent role in the discussion of perturbations of black holes, and the question arises whether they are as significant as normal modes are for self-adjoint systems, such as harmonic oscillators. They can be significant in two ways: Individual modes may dominate the time evolution of some perturbation, and a whole set of them could be used to completely describe this time evolution. It is known that quasinormal modes of black holes have the first property, but not the second. It has recently been suggested that a discontinuity in the underlying system would make the corresponding set of quasinormal modes complete. We therefore turn the Regge-Wheeler potential, which describes perturbations of Schwarzschild black holes, into a series of step potentials, hoping to obtain a set of quasinormal modes which shows both of the above properties. This hope proves to be futile, though: The resulting set of modes appears to be complete, but it no longer contains any individual mode which is directly obvious in the time evolution of initial data. Even worse, the quasinormal frequencies obtained in this way seem to be extremely sensitive to very small changes in the underlying potential. The question arises whether, and how, it is possible to make any definite statements about the significance of quasinormal modes of black holes at all, and whether it could be possible to obtain a set of quasinormal modes with the desired properties in another way.

PACS number(s): 04.70.Bw, 97.60.Lf

Black hole structural instability



Black hole structural instability

PHYSICAL REVIEW D **101,** 104009 (2020)

Significance of black hole quasinormal modes: A closer look

Ramin G. Daghigh,[1,2] Michael D. Green,[3] and Jodin C. Morey[4]

[1] *William I. Fine Theoretical Physics Institute, University of Minnesota, Minneapolis, Minnesota 55455, USA*

[2] *Natural Sciences Department, Metropolitan State University, Saint Paul, Minnesota 55106, USA*

[3] *Mathematics and Statistics Department, Metropolitan State University, Saint Paul 55106, Minnesota, USA*

[4] *School of Mathematics, University of Minnesota, Minneapolis, Minnesota 55455, USA*

(Received 28 February 2020; accepted 21 April 2020; published 5 May 2020)

It is known that approximating the Regge-Wheeler potential with step functions significantly modifies the Schwarzschild black hole quasinormal mode spectrum. Surprisingly, this change in the spectrum has little impact on the ringdown waveform. We examine whether this issue is caused by the jump discontinuities and/or the piecewise constant nature of step functions. We show that replacing the step functions with a continuous piecewise linear function does not qualitatively change the results. However, in contrast to previously published results, we discover that the ringdown waveform can be approximated to arbitrary precision using either step functions or a piecewise linear function. Thus, this approximation process provides a new mathematical tool to calculate the ringdown waveform. In addition, similar to normal modes, the quasinormal modes of the approximate potentials seem to form a complete set that describes the entire time evolution of the ringdown waveform. We also examine smoother approximations to the Regge-Wheeler potential, where the quasinormal modes can be computed exactly, to better understand how different portions of the potential impact various regions of the quasinormal mode spectrum.

DOI: 10.1103/PhysRevD.101.104009

Black hole structural instability

PHYSICAL REVIEW D **103,** 024019 (2021)

Asymptotical quasinormal mode spectrum for piecewise approximate effective potential

Wei-Liang Qian,[1,2,3,*] Kai Lin,[4,3] Cai-Ying Shao,[5] Bin Wang,[1] and Rui-Hong Yue[1]

[1]*Center for Gravitation and Cosmology, College of Physical Science and Technology, Yangzhou University, 225009, Yangzhou, China*
[2]*Institute for Theoretical Physics and Cosmology, Zhejiang University of Technology, 310032 Hangzhou, China*
[3]*Escola de Engenharia de Lorena, Universidade de São Paulo, 12602-810 Lorena, SP, Brazil*
[4]*Hubei Subsurface Multi-scale Imaging Key Laboratory, Institute of Geophysics and Geomatics, China University of Geosciences, 430074 Wuhan, Hubei, China*
[5]*MOE Key Laboratory of Fundamental Physical Quantities Measurement, Hubei Key Laboratory of Gravitation and Quantum Physics, PGMF, and School of Physics, Huazhong University of Science and Technology, 430074 Wuhan, Hubei, China*

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It was pointed out that the black hole quasinormal modes resulting from a piecewise approximate potential are drastically distinct from those pertaining to the original black hole metric. In particular, instead of lining up parallel to the imaginary axis, the spectrum is found to stretch out along the real axis. In this work, we prove that if there is a single discontinuity in the effective potential, no matter how insignificant it is, the asymptotic behavior of the quasinormal modes will be appreciably modified. Besides showing numerical evidence, we give analytical derivations to support the above assertion even when the discontinuity is located significantly further away from the maximum of the potential and/or the size of the step is arbitrarily small. Moreover, we discuss the astrophysical significance of the potential implications in terms of the present findings.

DOI: 10.1103/PhysRevD.103.024019

Black hole structural instability

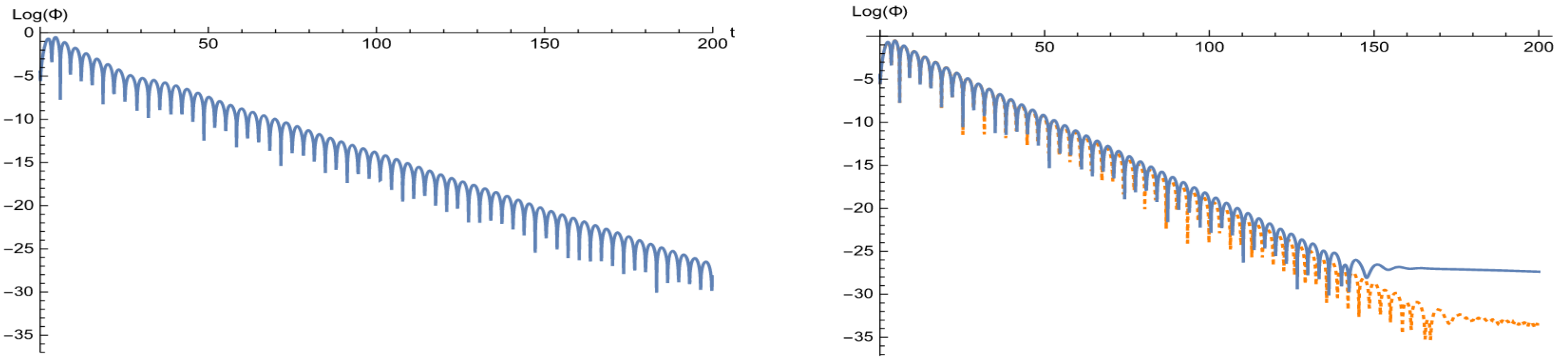


FIG. 3. Resultant temporal evolutions measured by an observer located at $r_* = 0$ for some initial Gaussian distribution centered at $r_* = 3$ with a width $\sigma = 1$. The blue solid curves shown in the left and right plots correspond to the cases where the "cut" and "step" defined in Eqs. (4) and (6) are located at $r_* \sim r = 10$. While the initial oscillations for $t \lesssim 15$ are identical for the two cases, a late-time tail for $t \gtrsim 150$ is observed for the potential with the "step." As a comparison, the results for the original Regge-Wheeler potential are also represented by the orange dotted curves in the right plot.

QIAN, LIN, SHAO, WANG, and YUE PHYS. REV. D **103,** 024019 (2021)

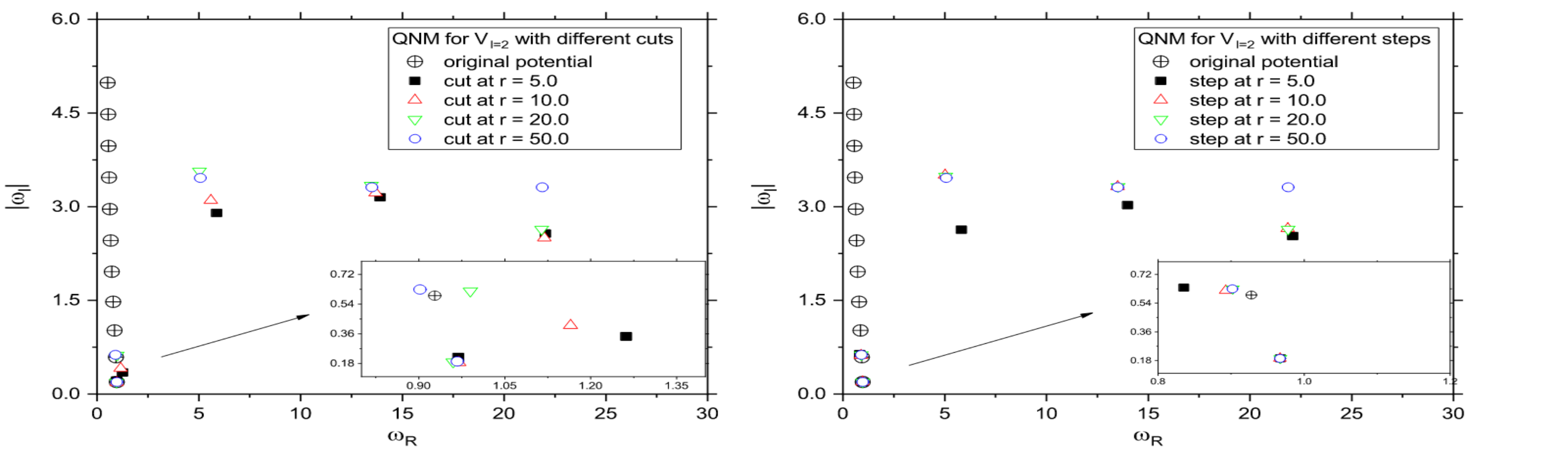


FIG. 4. Resultant low-lying quasinormal modes obtained from the potentials given in Fig. 2. The left and right plots show the two groups of potentials given in Eqs. (4) and (6).

Black hole structural instability

- No matter how weak it is, a discontinuity potentially leads to drastic modifications to higher overtones

- The low-lying modes seem less affected, and therefore the time-domain waveform remains mostly intact

Black hole structural instability

PHYSICAL REVIEW X **11**, 031003 (2021)

Featured in Physics

Pseudospectrum and Black Hole Quasinormal Mode Instability

José Luis Jaramillo,[1] Rodrigo Panosso Macedo,[2] and Lamis Al Sheikh[1]

[1]*Institut de Mathématiques de Bourgogne (IMB), UMR 5584, CNRS, Université de Bourgogne Franche-Comté, F-21000 Dijon, France*

[2]*School of Mathematical Sciences, Queen Mary, University of London, Mile End Road, London E1 4NS, United Kingdom*

(Received 14 April 2020; accepted 3 May 2021; published 6 July 2021)

We study the stability of quasinormal modes (QNM) in asymptotically flat black hole spacetimes by means of a pseudospectrum analysis. The construction of the Schwarzschild QNM pseudospectrum reveals the following: (i) the stability of the slowest-decaying QNM under perturbations respecting the asymptotic structure, reassessing the instability of the fundamental QNM discussed by Nollert [H. P. Nollert, *About the Significance of Quasinormal Modes of Black Holes*, Phys. Rev. D **53**, 4397 (1996)] as an "infrared" effect; (ii) the instability of all overtones under small-scale ("ultraviolet") perturbations of sufficiently high frequency, which migrate towards universal QNM branches along pseudospectra boundaries, shedding light on Nollert's pioneer work and Nollert and Price's analysis [H. P. Nollert and R. H. Price, *Quantifying Excitations of Quasinormal Mode Systems*, J. Math. Phys. (N.Y.) **40**, 980 (1999)]. Methodologically, a compactified hyperboloidal approach to QNMs is adopted to cast QNMs in terms of the spectral problem of a non-self-adjoint operator. In this setting, spectral (in)stability is naturally addressed through the pseudospectrum notion that we construct numerically via Chebyshev spectral methods and foster in gravitational physics. After illustrating the approach with the Pöschl-Teller potential, we address the Schwarzschild black hole case, where QNM (in)stabilities are physically relevant in the context of black hole spectroscopy in gravitational-wave physics and, conceivably, as probes into fundamental high-frequency spacetime fluctuations at the Planck scale.

DOI: 10.1103/PhysRevX.11.031003

Subject Areas: Astrophysics, Gravitation, Interdisciplinary Physics, Optics

Black hole structural instability

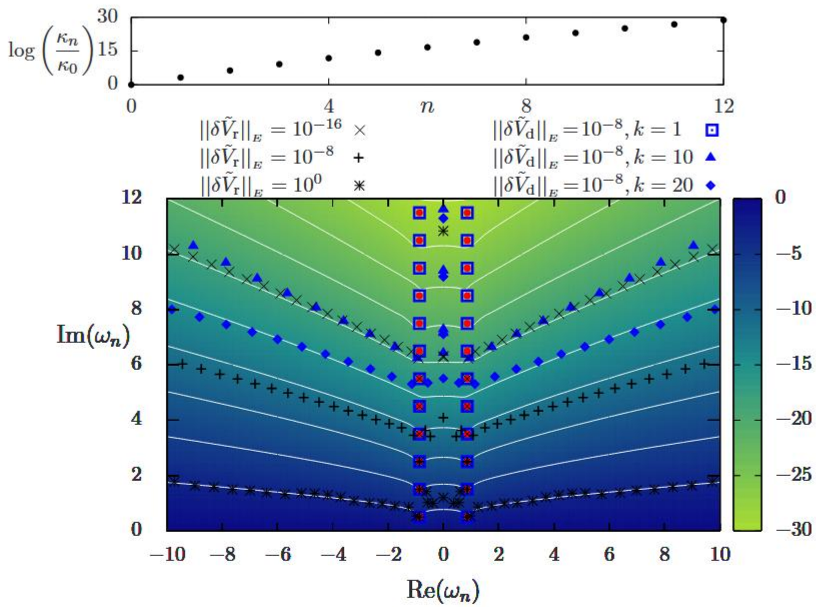


FIG. 9. QNM spectral instability of Pöschl-Teller potential. Combination of Figs. 3, 5 and 7, corresponding to three independent calculations, respectively: condition numbers ratios κ_n/κ_0 (*top panel*), pseudospectrum and perturbed QNM spectra (*bottom panel*). The bottom pannel demonstrates the high-frequency nature of the spectral instability, as well as the migration of Pöschl-Teller QNMs towards pseudospectrum contour lines under high-frequency perturbations.

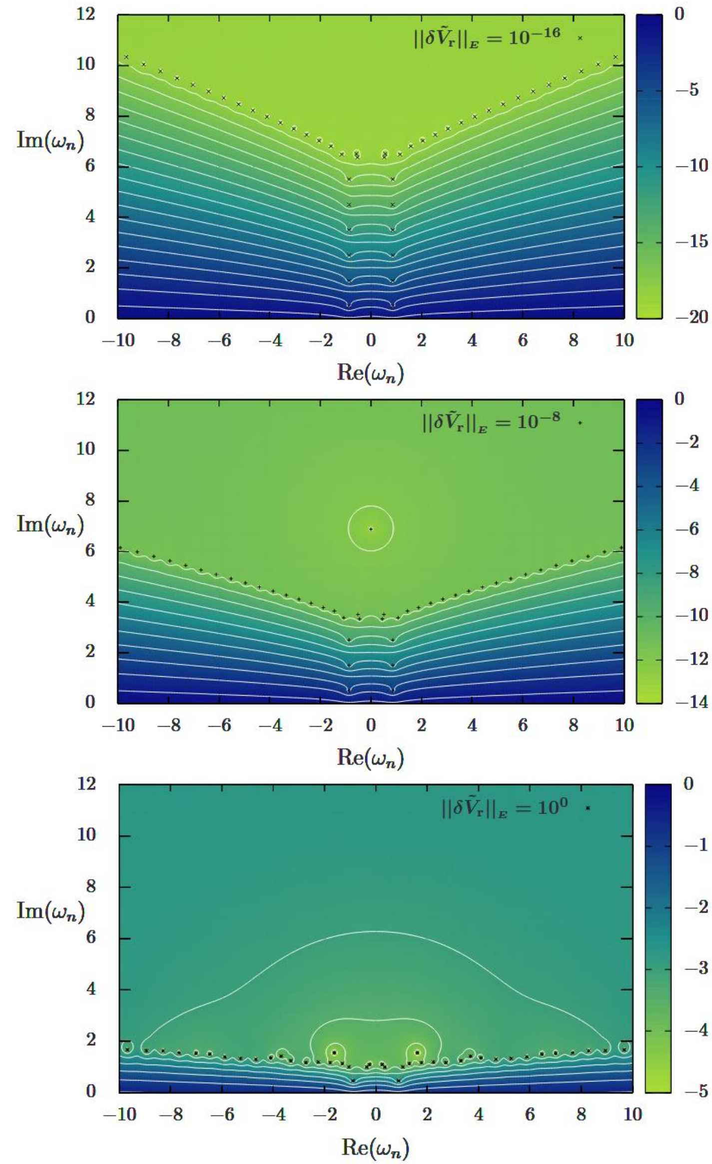


FIG. 11. Pseudospectra of Pöschl-Teller under random perturbations $\delta\tilde{V}_r$ of increasing norm, demonstrating the "regularizing" effect of random perturbations: pseudospectra sets σ^ϵ bounded by that "contour line" reached by perturbed QNMs become "flat", a signature of improved analytic behaviour of the resolvent, as illustrated in Fig. 6. Pseudospectra sets not attained by the perturbation remain unchanged. Regularization of $R_{L+\delta L}(\omega)$ increases as $||\delta\tilde{V}_r||_E$ grows.

Black hole structural instability

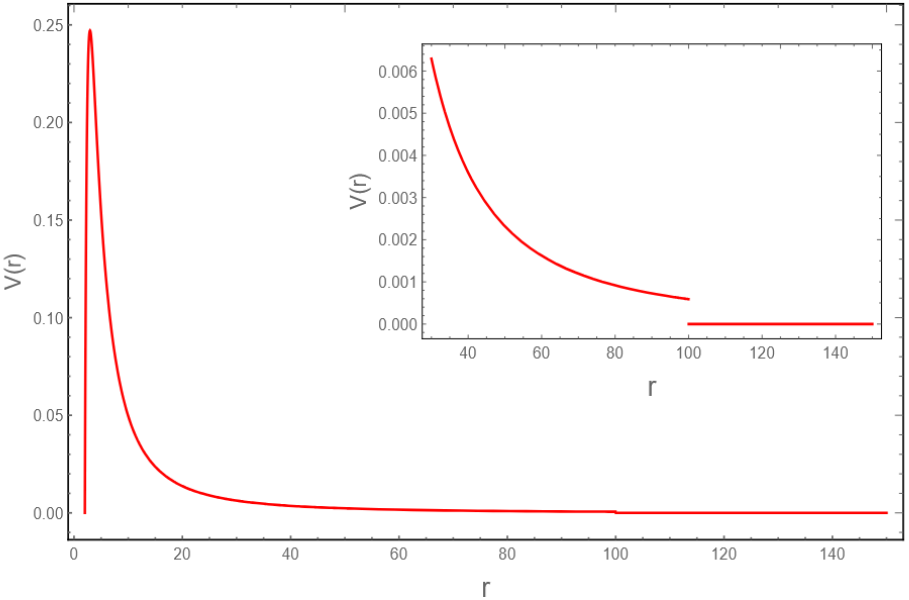
- Structural instability is different from the stability of individual states
- No matter how weak the metric perturbation is, it potentially leads to drastic modifications to higher overtones
- The low-lying modes seem less affected, and therefore the time-domain waveform remains mostly intact

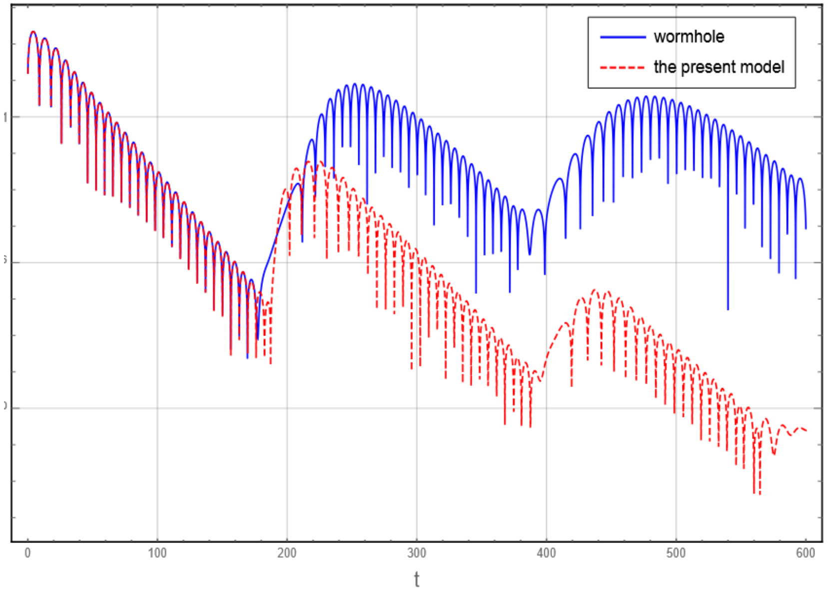
Deformed high overtones furnish a recipe for GW echoes

- Bouncing back and forth between the maxima of the effective potential

- Analysis based on QNM spectrum (arXiv:1711.00391)
- Analysis based on Green's function (arXiv:1706.06155)
- Analysis based on the poles of the Green's function (arXiv:2104.11912)

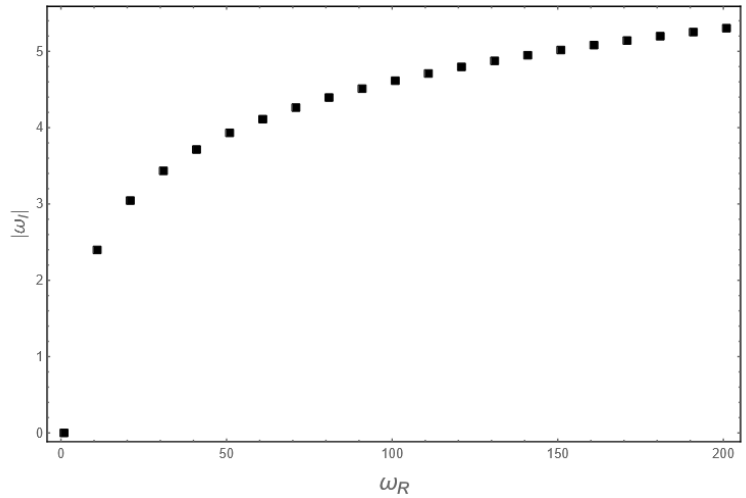
Echoes and poles of Green's function owing to discontinuity





arXiv:2104.11912v4

Echoes and poles of Green's function owing to discontinuity



$$
\begin{aligned}
&G(\omega,x,x') \to \tilde{G}(\omega,x,x') \\
&= \frac{1}{W(\tilde{f}_+,f_-)} f_-(\omega,x_<)\tilde{f}_+(\omega,x_>) \\
&= G(\omega,x,x') + \frac{\mathcal{R}}{A_{\rm in}e^{-2i\omega x_s} - \mathcal{R}A_{\rm out}}\frac{1}{W(\omega)} f_-(\omega,x_<)f_-(\omega,x_>) \\
&= G(\omega,x,x') + \frac{\mathcal{R}}{A_{\rm in}e^{-2i\omega x_s} - \mathcal{R}A_{\rm out}}\frac{1}{W(\omega)} f_-(\omega,x)f_-(\omega,x'),
\end{aligned}
$$

Instability in the fundamental mode of Regge-Wheeler potential

Destabilizing the Fundamental Mode of Black Holes: The Elephant and the Flea

Mark Ho-Yeuk Cheung,[1] Kyriakos Destounis,[2] Rodrigo Panosso Macedo,[3,4] Emanuele Berti,[1] and Vitor Cardoso[5,3]

[1] *Department of Physics and Astronomy, Johns Hopkins University, 3400 North Charles Street, Baltimore, Maryland, 21218, USA*

[2] *Theoretical Astrophysics, IAAT, University of Tübingen, 72076 Tübingen, Germany*

[3] *CENTRA, Departamento de Física, Instituto Superior Técnico–IST, Universidade de Lisboa–UL, Avenida Rovisco Pais 1, 1049 Lisboa, Portugal*

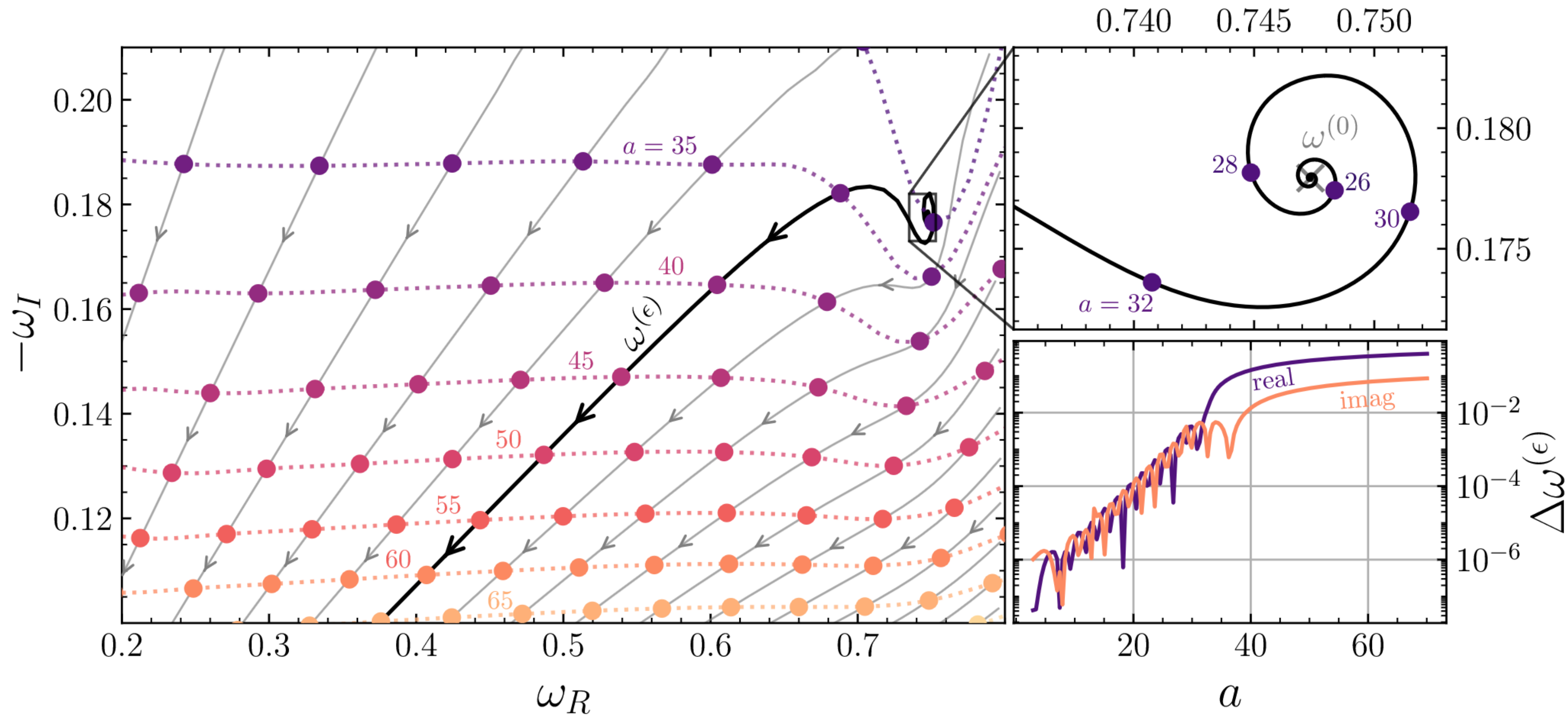
[4] *STAG Research Centre, University of Southampton, University Road SO17 1BJ, Southampton, United Kingdom*

[5] *Niels Bohr International Academy, Niels Bohr Institute, Blegdamsvej 17, 2100 Copenhagen, Denmark*

(Dated: March 29, 2022)

Recent work applying the notion of pseudospectrum to gravitational physics showed that the quasinormal mode spectrum of black holes is unstable, with the possible exception of the longest-lived (fundamental) mode. The fundamental mode dominates the expected signal in gravitational wave astronomy, and there is no reason why it should have privileged status. We compute the quasinormal mode spectrum of two model problems where the Schwarzschild potential is perturbed by a small "bump" consisting of either a Pöschl-Teller potential or a Gaussian, and we show that the fundamental mode is destabilized under generic perturbations. We present phase diagrams and study a simple double-barrier toy problem to clarify the conditions under which the spectral instability occurs.

Instability in the fundamental mode of Regge-Wheeler potential



Black hole structural instability

- Structural instability is different from the stability of individual states
- No matter how weak the metric perturbation is, it potentially leads to drastic modifications to higher overtones and low-lying modes
- The time-domain waveform remains mostly intact*

Emergence of spectral instability

- Analytically accessible example (e.g. truncated PT & disjoint potentials)
- Truncated PT
- asymptotical properties of the hypergeometric function
- extensively investigated by other authors (arXiv:1004.2539, 1007.4039, 2009.11627, 2406.10782)

- Disjoint square barrier potential
- analytic forms of the transmission matrix that provide a straightforward picture
- employed by various authors (arXiv: 2210.01724, 2407.15191, 2407.20144, 2409.17026)

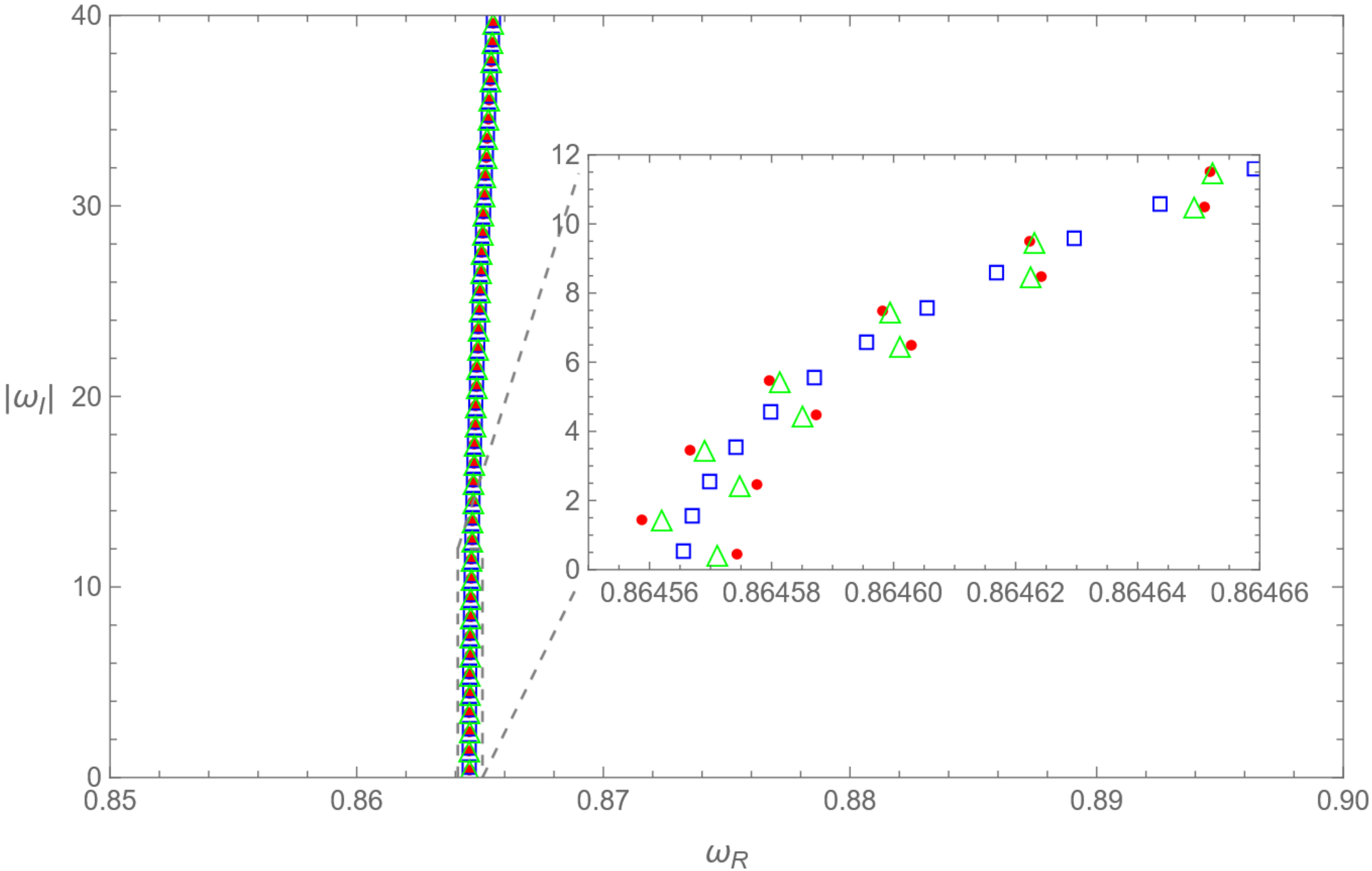
- Demonstrate explicitly and dynamically how spectral instability emerges as the perturbation (implemented by a discontinuity) moves away from the black hole

- To implement this, the matrix method is generalized to the hyperboloidal coordinates on the Chebyshev grid

A dynamic picture of spectral instability

- To implement this, the matrix method is generalized to the hyperboloidal coordinates on the Chebyshev grid
- Matrix method is an approach reminiscent to the continued fraction method where the expansion is carried out at different nodes on a grid
- Chebyshev grid is an optimal choice for polynomial expansion where Runge instability is significant suppressed
- Hyperboloidal coordinates is an alternative but natural choice of null infinity as the bound for QNM master equation instead of conventional spatial infinity where the wave function becomes divergent

Emergence of spectral instability



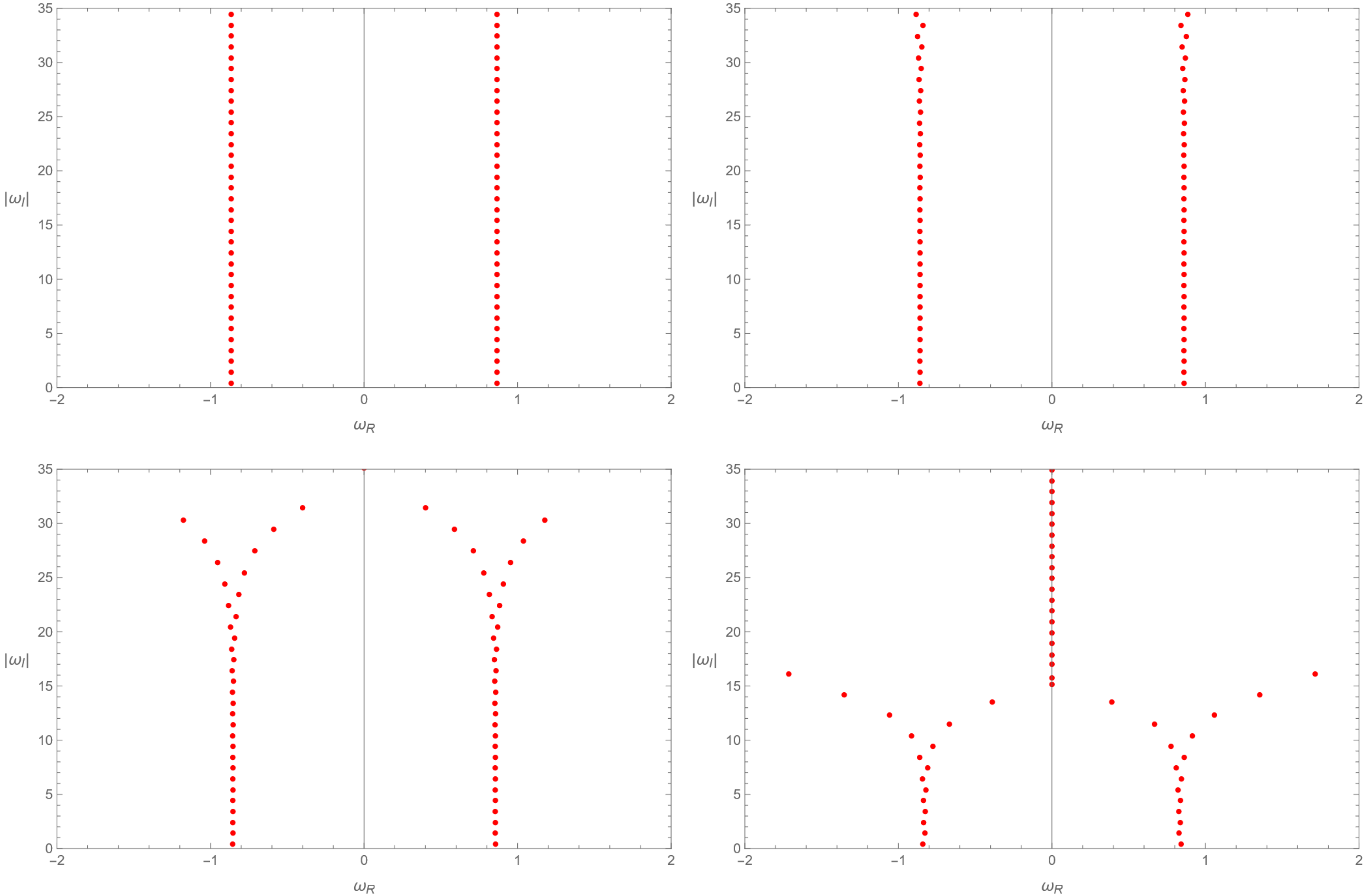
A “contradiction” in the existing literature about QNM in the perturbed PT potential

- Visser’s perturbative results (arXiv:1004.2539, arXiv:1007.4039)

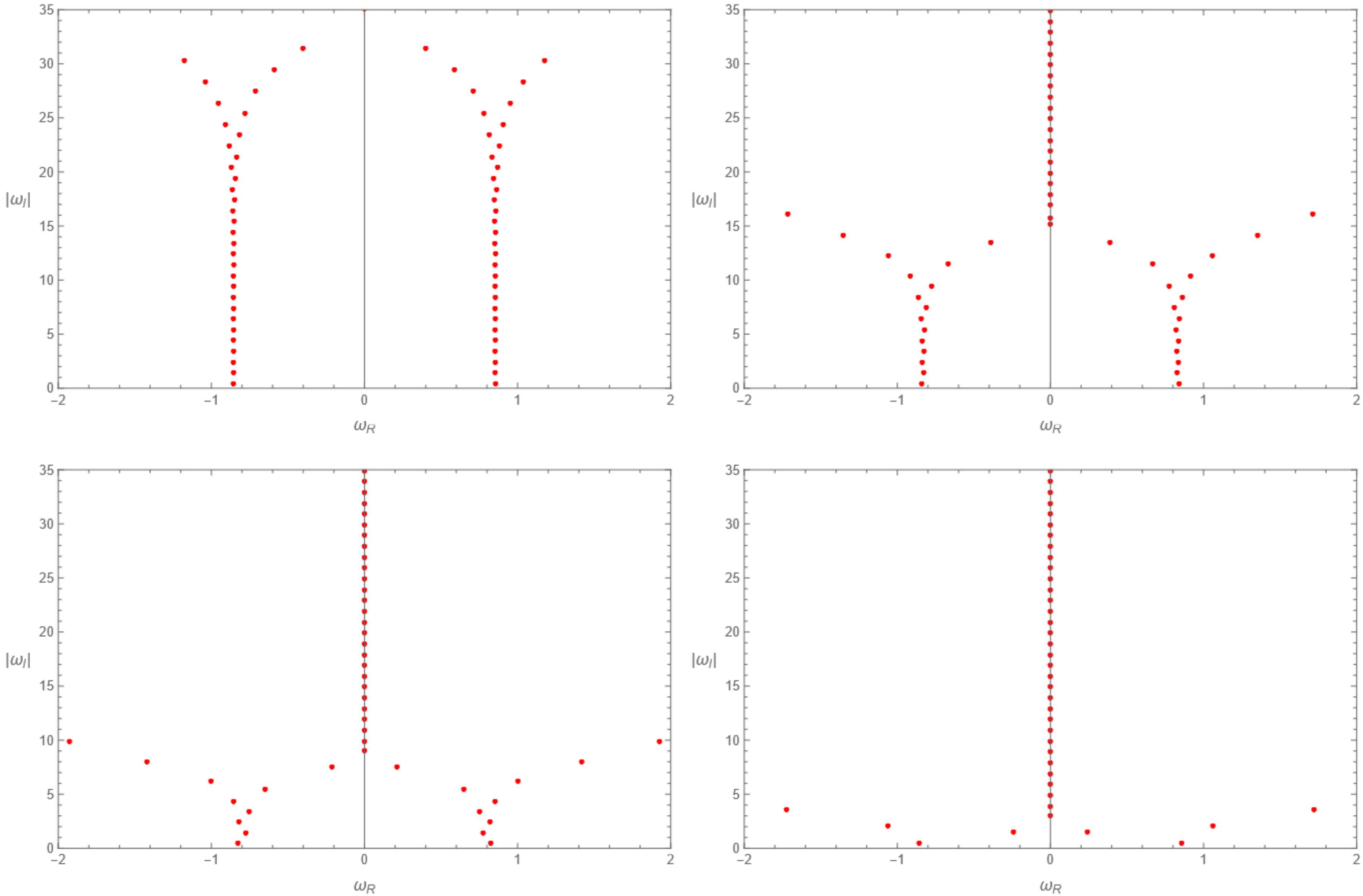
- Analytic calculations of asymptotic behavior (arXiv:2009.11627) in the modified PT potential

- Observation of bifurcation in the QNM spectrum

- Dynamic evolution of high overtones from *parallel to the imaginary frequency axis* to *parallel to the real frequency axis*



$|\omega_I|$
ω_R



35
30
25
20
15
10
5
0
−2
−1
0
1
2
$|\omega_I|$
ω_R

Instability in the fundamental mode of Regge-Wheeler potential

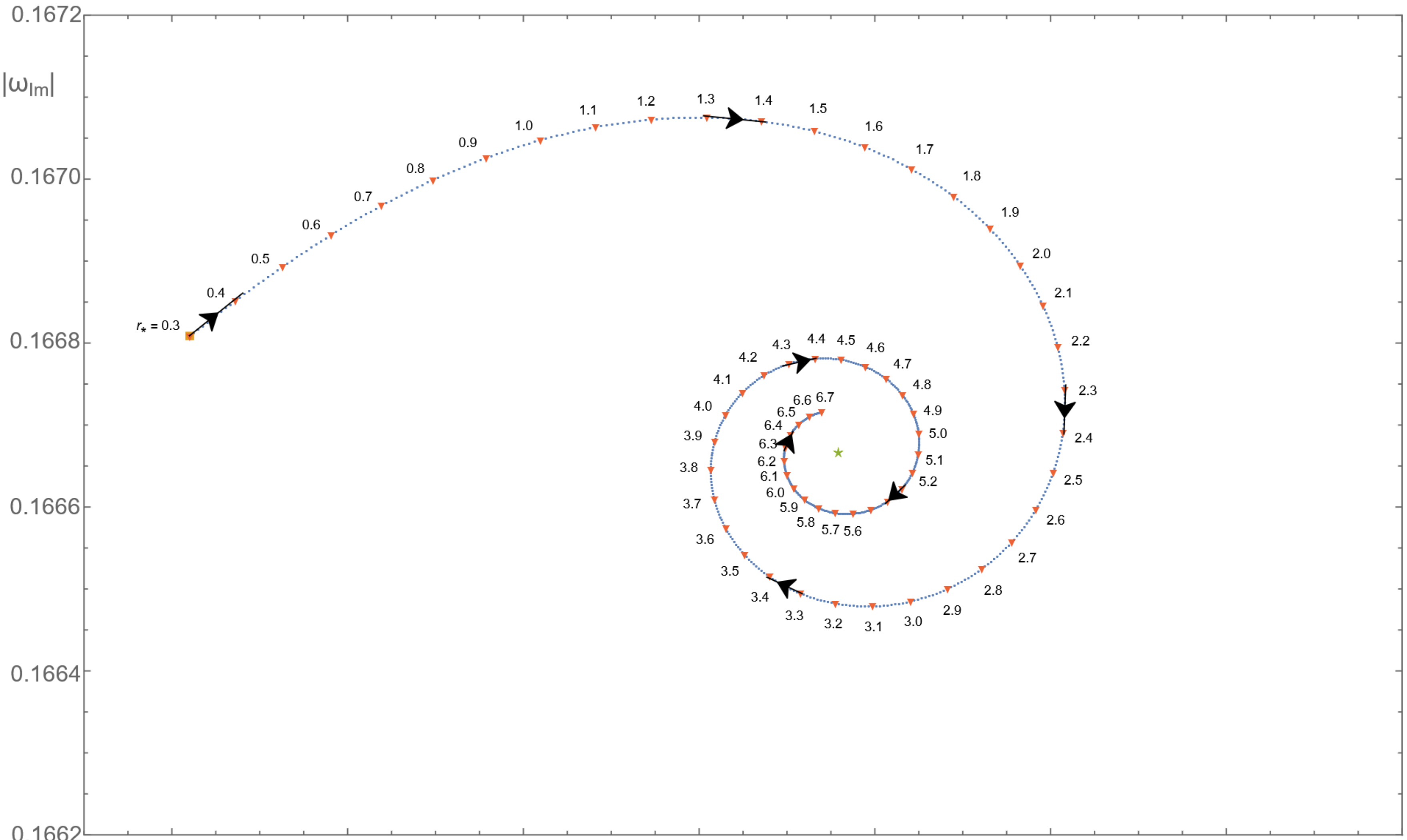
- Disjoint effective potentials have been utilized as a toy model to illustrate such an instability (arXiv: 2210.01724, 2407.15191, 2407.20144, 2409.17026)

- Is it reasonable as an approximation?
- Is it physically plausible?
- Does it always imply instability?
- Observational implications?

Instability in the fundamental mode of Regge-Wheeler potential

- An analytic derivation can be given from a rather general context
- The fundamental mode of truncated PT potential was found to be stable, in contrast with most studies in the literature
- Disjoint effective potential is always unstable, independent of specific potential form
- Analytic results are in good agreement with numerical calculations

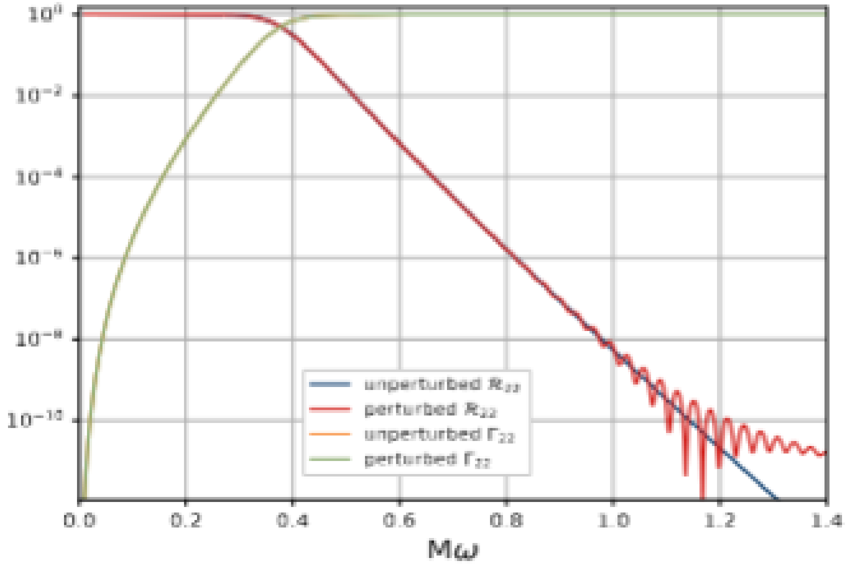
Instability in the fundamental mode of Regge-Wheeler potential

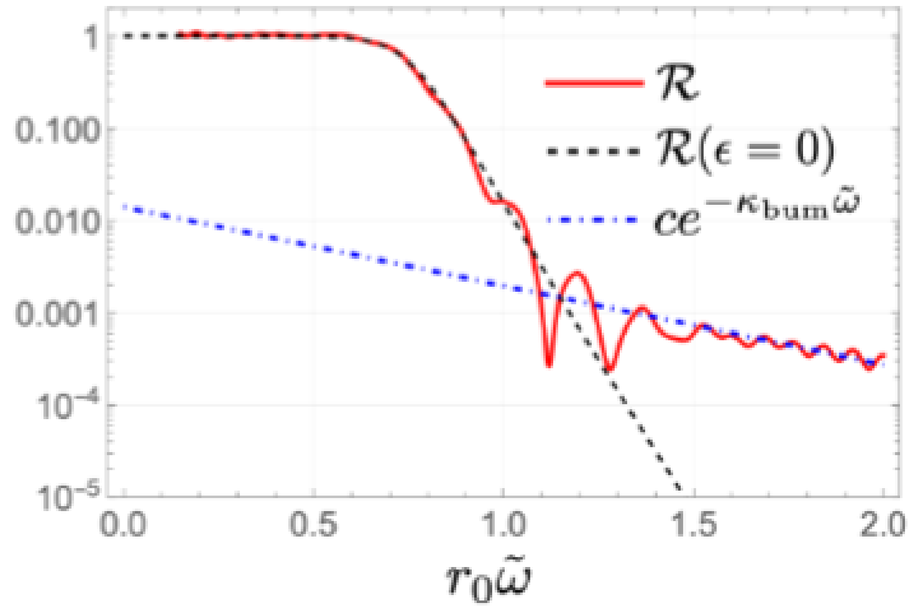


Greybody factor as stable observables

- Rosato *et al.* (arXiv: 2406.01692) and Oshita *et al.* (arXiv:2406.04525) pointed out that graybody factors are stable observables at relatively high frequencies

Greybody factor as stable observables





Rosato *et al.* (arXiv: 2406.01692) and Oshita *et al.* (arXiv:2406.04525)

Greybody factor as stable observables

- Rosato *et al.* (arXiv: 2406.01692) and Oshita *et al.* (arXiv:2406.04525) pointed out that graybody factors are stable observables at relatively high frequencies

- When viewed as a scattering problem, the graybody factor can be viewed to receive contributions from different partial waves while sitting on top of the background eikonal limit

Greybody factor as stable observables

$$\sigma_{\text{abs}}(\omega) \equiv \int \frac{d\sigma_{\text{abs}}}{d\Omega} d\Omega = \sigma_{\text{abs}}^{\text{RP}}(\omega) + \sigma_{\text{abs}}^{\text{BG}}(\omega),$$

$$\sigma_{\text{abs}}^{\text{RP}}(\omega) = -\frac{4\pi^2}{\omega^2} \text{Re}\left[\sum_{n=0}^{\infty} \frac{\lambda_n(\omega)\gamma_n(\omega)e^{i\pi\ell_n(\omega)}}{\sin \pi\ell_n(\omega)}\right]$$

$$r_n \equiv \text{Res}[e^{i\pi(\ell+1)} R_\ell(\omega)]_{\ell=\ell_n}.$$

$$f(\omega, \theta) = f^{\text{RP}}(\omega, \theta) + f^{\text{BG}}(\omega, \theta),$$

$$\Gamma_\ell(\omega) \equiv \Gamma_{\lambda-1/2}(\omega) = |T_\ell(\omega)|^2,$$

Greybody factor as stable observables

- Rosato *et al.* (arXiv: 2406.01692) and Oshita *et al.* (arXiv:2406.04525) pointed out that graybody factors are stable observables at relatively high frequencies

- When viewed as a scattering problem, the graybody factor can be viewed to receive contributions from different partial waves while sitting on top of the background eikonal limit

- The greybody factor is essentially the squared module of transmission amplitude, therefore, it seems that the stability of the greybody factor resides in that of Regge poles

Greybody factor as stable observables

$$\Gamma_{\ell}(\omega) \equiv \Gamma_{\lambda-1/2}(\omega) = |T_{\ell}(\omega)|^2 ,$$

Greybody factor as stable observables

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- The greybody factor is essentially the squared module of transmission amplitude, therefore, it seems that the stability of the greybody factor resides in that of Regge poles

- However, Regge poles are not really “stable” and shown to subject to spectral instability

Scattering amplitude constituted by the underlying Regge poles

PHYSICAL REVIEW LETTERS **131,** 111401 (2023)

From Black Hole Spectral Instability to Stable Observables

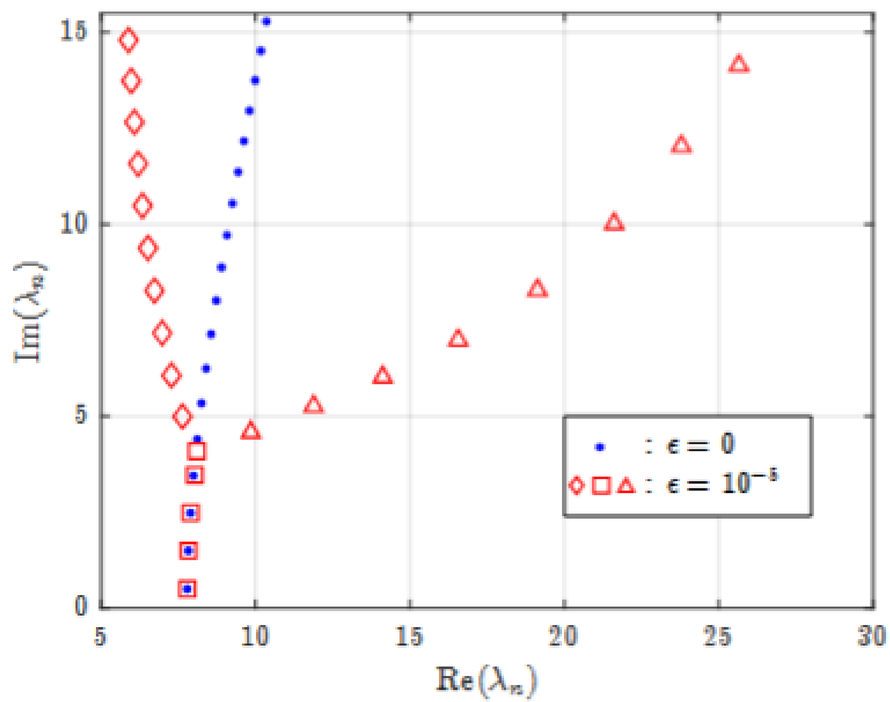
Théo Torres*
Aix Marseille Université, CNRS, Centrale Marseille, LMA UMR 7031, Marseille, France

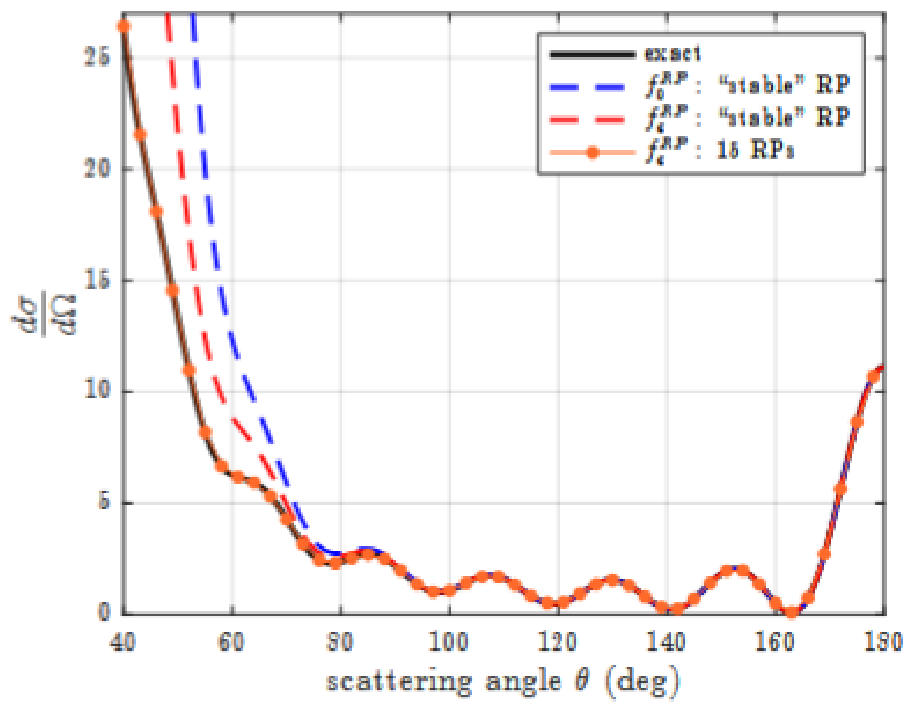
(Received 4 May 2023; revised 16 June 2023; accepted 8 August 2023; published 11 September 2023)

The quasinormal mode spectrum of black holes is unstable under small perturbation of the potential and has observational consequences in time signals. Such signals might be experimentally difficult to observe and probing this instability will be a technical challenge. Here, we investigate the spectral instability of time-independent data. This leads us to study the Regge poles (RPs), the counterparts to the quasinormal modes in the complex angular momentum plane. We present evidence that the RP spectrum is unstable but that not all overtones are affected equally by this instability. In addition, we reveal that behind this spectral instability lies an underlying structure. The RP spectrum is perturbed in such a way that one can still recover *stable* scattering quantities using the complex angular momentum approach. Overall, the study proposes a novel and complementary approach on the black hole spectral instability phenomena that allows us to reveal a surprising and unexpected mechanism at play that protects scattering quantities from the instability.

DOI: 10.1103/PhysRevLett.131.111401

Greybody factor as stable observables






WHY?

A dynamic picture of spectral instability

- To answer this, the matrix method is generalized to evaluate Regge poles in the hyperboloidal coordinates on the Chebyshev grid

- We evaluate Regge poles, transmission coefficients, and the scattering amplitude, graybody factors, and explore their dependence on frequency and other parameters.

Hyperboloidal coordinates

PHYSICAL REVIEW D **83,** 127502 (2011)

A geometric framework for black hole perturbations

Anıl Zenginoğlu

Theoretical Astrophysics, California Institute of Technology, Pasadena, California, USA

(Received 15 February 2011; published 17 June 2011)

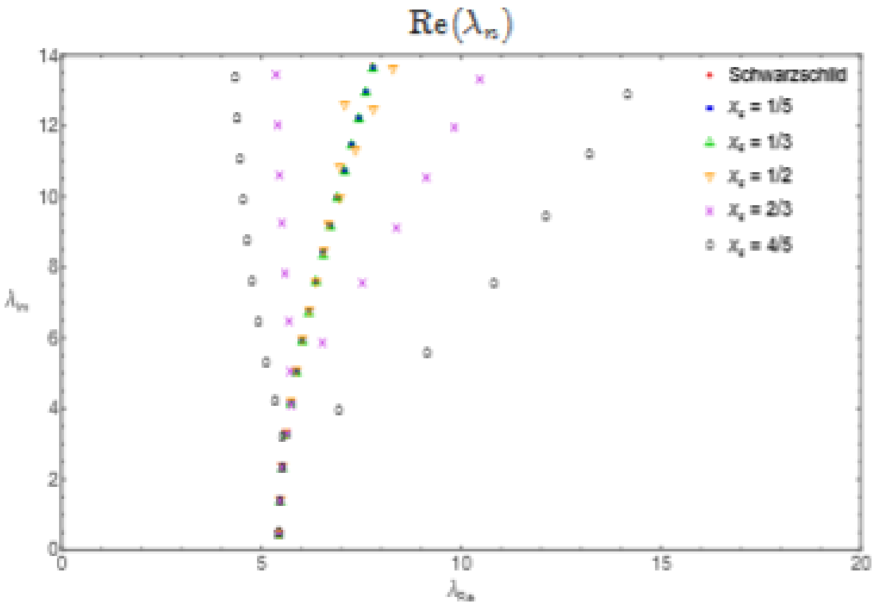
Black hole perturbation theory is typically studied on time surfaces that extend between the bifurcation sphere and spatial infinity. From a physical point of view, however, it may be favorable to employ time surfaces that extend between the future event horizon and future null infinity. This framework resolves problems regarding the representation of quasinormal mode eigenfunctions and the construction of short-ranged potentials for the perturbation equations in frequency domain.

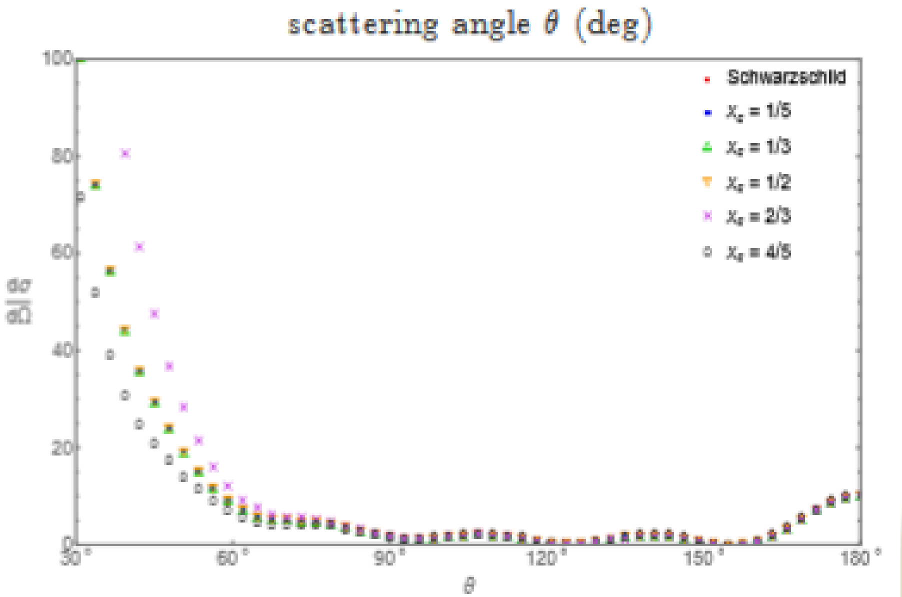
DOI: 10.1103/PhysRevD.83.127502 PACS numbers: 04.25.Nx, 04.70.Bw, 04.20.Ha

	ω	x_c	1/5	1/3	1/2	2/3	4/5
IMM	0.1	$\mathrm{Re}(\lambda_0)$	1.774253	1.693898	1.617204	1.557917	1.528656
		$\mathrm{Im}(\lambda_0)$	0.285414	0.182655	0.132764	0.110739	0.102096
		$\mathrm{Re}(\lambda_1)$	0.451043	0.380904	0.370980	0.440004	0.575100
		$\mathrm{Im}(\lambda_1)$	3.598320	2.390483	1.503480	0.874645	0.526054
	2.0	$\mathrm{Re}(\lambda_0)$	6.716861	5.671341	5.256377	5.477237	5.788201
		$\mathrm{Im}(\lambda_0)$	1.858302	1.204032	0.661783	0.323130	0.594422
		$\mathrm{Re}(\lambda_1)$	6.928930	5.618690	4.990127	5.071166	5.334647
		$\mathrm{Im}(\lambda_1)$	4.735250	3.198286	2.044238	1.064024	0.595809
CFM	0.1	$\mathrm{Re}(\lambda_0)$	1.774253	1.693899	1.617216	1.558007	1.528872
		$\mathrm{Im}(\lambda_0)$	0.285414	0.182655	0.132764	0.110729	0.102096
		$\mathrm{Re}(\lambda_1)$	0.451043	0.380904	0.370986	0.440233	0.575376
		$\mathrm{Im}(\lambda_1)$	3.598320	2.390481	1.503475	0.875028	0.528817
	2.0	$\mathrm{Re}(\lambda_0)$	6.716861	5.671342	5.256464	5.476614	5.786688
		$\mathrm{Im}(\lambda_0)$	1.858301	1.204025	0.661678	0.324511	0.593437
		$\mathrm{Re}(\lambda_1)$	6.928930	5.618684	4.990179	5.071176	5.334340
		$\mathrm{Im}(\lambda_1)$	4.735252	3.198287	2.044712	1.066662	0.596942

	ω	x_c	1/5	1/3	1/2	2/3	4/5
IMM	0.1	$\mathrm{Re}(\lambda_0)$	1.360288	1.268653	1.174931	1.146382	1.301967
		$\mathrm{Im}(\lambda_0)$	0.094145	0.116269	0.176222	0.307970	0.521280
		$\mathrm{Re}(\lambda_1)$	0.697955	0.766626	0.903650	1.141025	1.542527
		$\mathrm{Im}(\lambda_1)$	0.764168	0.961770	1.202828	1.487517	1.830079
	2.0	$\mathrm{Re}(\lambda_0)$	5.340572	5.660748	6.582699	8.656515	12.905628
		$\mathrm{Im}(\lambda_0)$	0.664842	0.972551	1.344722	1.800766	2.388594
		$\mathrm{Re}(\lambda_1)$	5.359182	5.908920	7.120976	9.518427	14.122522
		$\mathrm{Im}(\lambda_1)$	1.924792	2.415844	3.041853	3.850981	4.904038
CFM	0.1	$\mathrm{Re}(\lambda_0)$	1.360256	1.268666	1.174929	1.146381	1.301967
		$\mathrm{Im}(\lambda_0)$	0.094148	0.116276*i*	0.176223	0.307971	0.521280
		$\mathrm{Re}(\lambda_1)$	0.698675	0.766448	0.903694	1.141281	1.542476
		$\mathrm{Im}(\lambda_1)$	0.763163	0.961685	1.202493	1.487527	1.830102
	2.0	$\mathrm{Re}(\lambda_0)$	5.340564	5.660748	6.582699	8.656515	12.905628
		$\mathrm{Im}(\lambda_0)$	0.664837	0.972551	1.344722	1.800766	2.388594
		$\mathrm{Re}(\lambda_1)$	5.359185	5.908902	7.120967	9.518454	14.122522
		$\mathrm{Im}(\lambda_1)$	1.924777	2.415850	3.041877	3.850958	4.904039

Greybody factor as stable observables

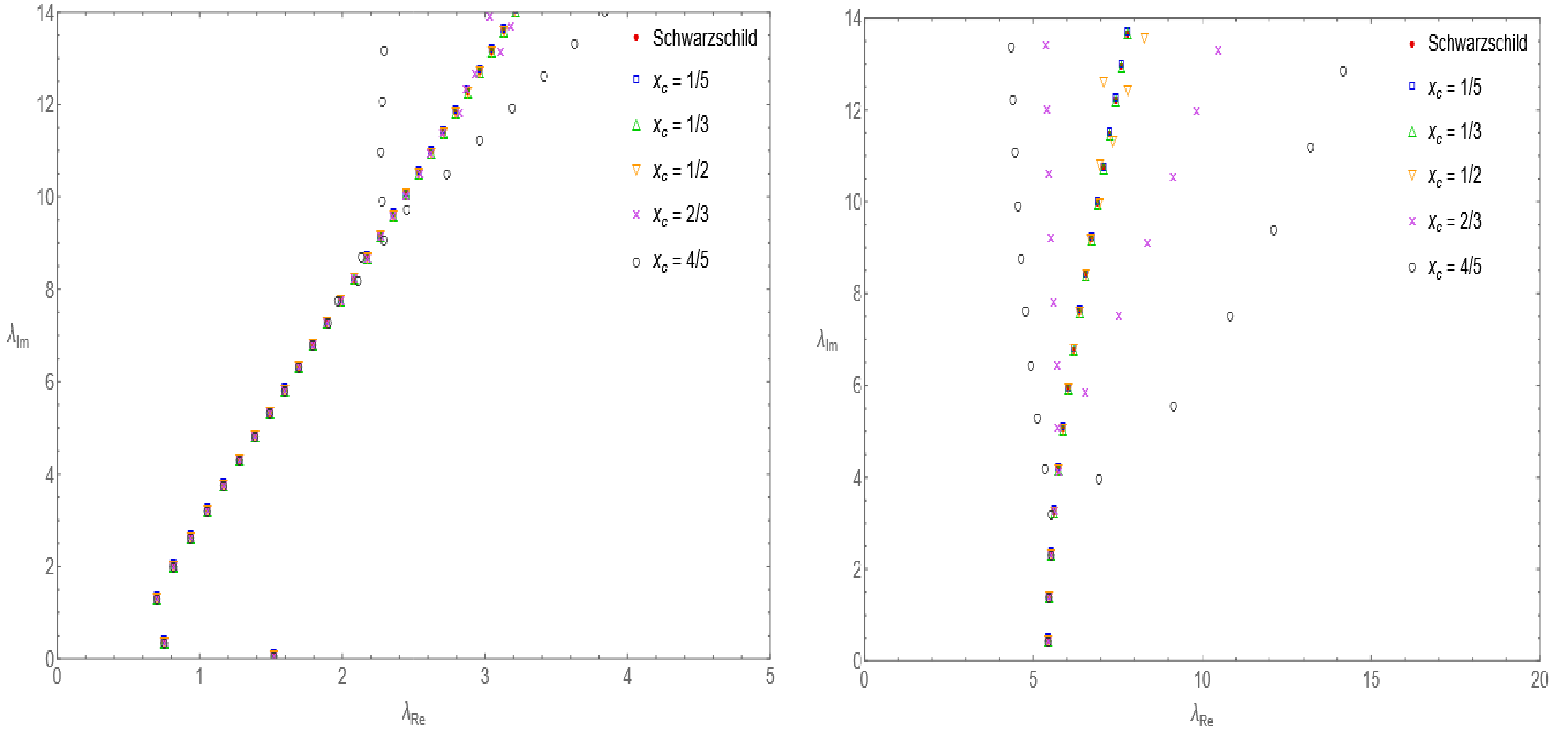




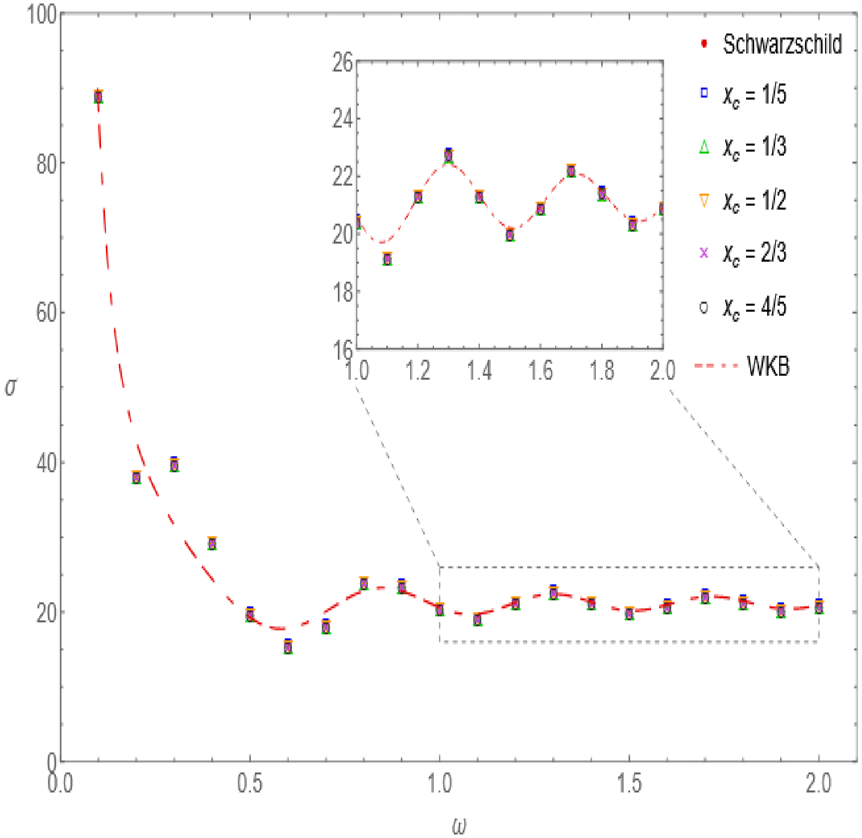
A dynamic picture of spectral instability

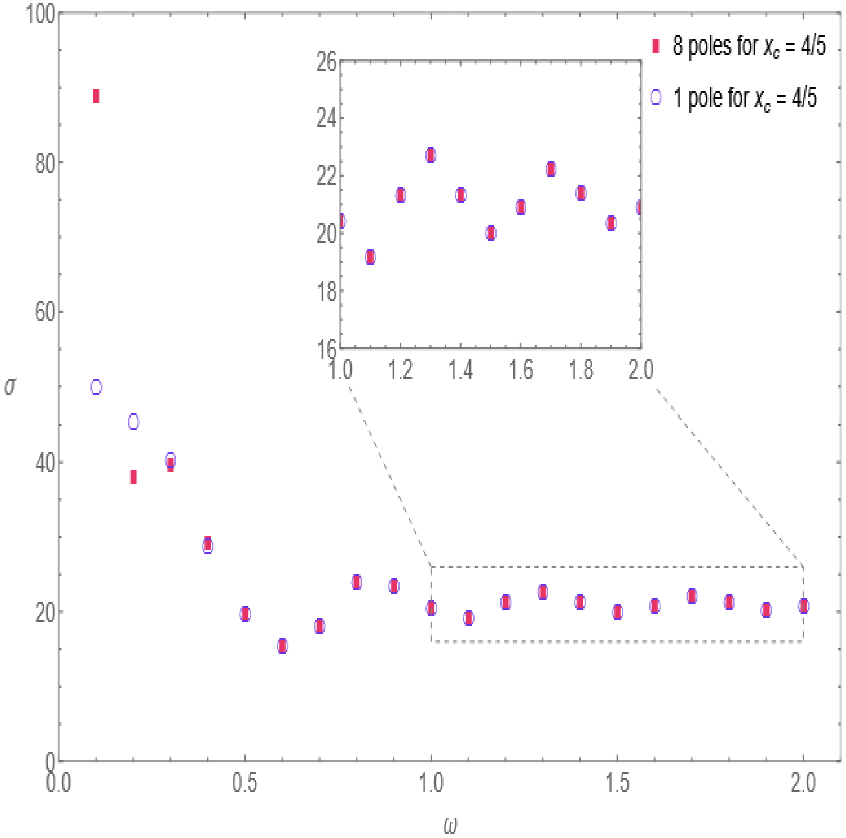
- At lower frequencies, the spectral instability is not a severe issue
- At higher frequencies, where the eikonal limit is largely valid, Regge poles are not supposed to be important

Greybody factor as stable observables



Greybody factor as stable observables





Concluding remarks

- QNMs and their stability, as generic properties of any dissipative system, play a pivotal role in the ongoing effort of GW astronomy
- The recent developments on spectral instability of QMNs and Regge poles, place this concept in extensive discussion in the context of black hole spectroscopy
- Its observational implication, IMHO, is not yet settled

Speculations

- Do time-domain stability or observables such as the greybody factor guarantee the success in extracting the underlying metric parameters?
- A complete analysis from simulated GW signals, that includes simulated data with realistic noise and are accompanied by TDI scheme using Bayesian/ML algorithms is warranted
- Reflectionless modes(?)

Thank you!

谢谢!

backup slides