Generalized harmonic gauge on compactified hyperboloidal slices

David Hilditch

CENTRA, IST Lisbon

Virtual Infinity Seminar, September 2024

- Class.Quant.Grav. 35 (2018) no.5, 055003, with E. Harms, M. Bugner, H. Rüter, B. Brügmann.
- Class.Quant.Grav. 36 (2019) with E. Gasperin.
- Class.Quant.Grav. 37 (2020) with E. Gasperin, S. Gautam, A. Vañó-Viñuales.
- Phys.Rev.D103, 084045 (2021) with S. Gautam, A. Vañó-Viñuales, S Bose.
- Class.Quant.Grav. 38 (2021) with M. Duarte, J. Feng, E. Gasperin.
- Class.Quant.Grav. 39 (2022) with M. Duarte, J. Feng, E. Gasperin.
- Class.Quant.Grav. 40 (2023) with M. Duarte, J. Feng, E. Gasperin.
- Phys.Rev.D108, 024067 (2023) with C. Peterson, S. Gautam, I. Rainho, A. Vañó-Viñuales.
- arXiv:2409.02994 (2024) with C. Peterson, S. Gautam, A. Vañó-Viñuales.

Wanted: Gravitational Waves at $\mathscr{I}^+$

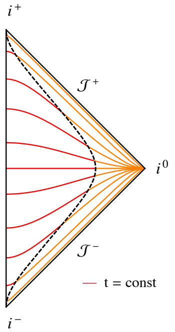
We are concerned with the *first principles* computation of gravitational waves at future null infinity.

State-of-the-art:

- Extrapolation.
- Characteristic-Extraction.

Wish-list:

- Well-posedness. Nice equations and solutions.
- Extension of strong-field setup.
- Proveably good numerics.



Timelike outer boundary. Vañó-Viñuales.

The weak-field

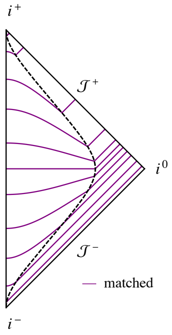
The wavezone is *weak*, so how is it a problem? Infinity is *really* big.

Fundamental ingredients:

- Compactify whilst resolving outgoing waves. Introduces blow-up quantities.
- Asymptotic Flatness: Metric decays near infinity.

Key to any computational strategy is the management of this competition.

Examples: CEFES. CCE/CCM.



CCM Cartoon. Vañó-Viñuales. 2015.

The weak-field

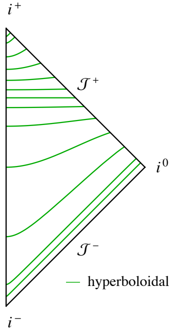
The wavezone is *weak*, so how is it a problem? Infinity is *really* big.

Fundamental ingredients:

- Compactify whilst resolving outgoing waves. Introduces blow-up quantities.
- Asymptotic Flatness: Metric decays near infinity.

Key to any computational strategy is the management of this competition.

Examples: CEFES. CCE/CCM.



Hyperboloidal foliation. Vañó-Viñuales.

Compactification: dual foliation formalism

General strategy: take good formulation of GR in tensor basis of $X^{\underline{\mu}}$, but work with independent variables $x^{\mu} = (t, x^i)$.

- Given hyperbolic system

$$\partial_T \mathbf{u} = \mathbf{A}^{\underline{p}} \partial_{\underline{p}} \mathbf{u} + \mathbf{S}.$$

- Change coordinates:

$$T = t + H(R),\ R = R(r),\ \theta^{\underline{A}} = \theta^{A}.$$

- Adjust equations via Jacobian, rescale variables.

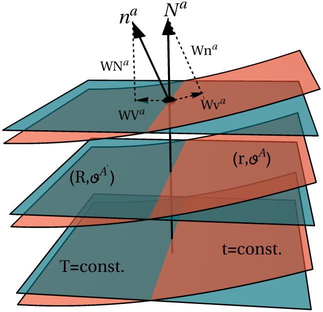


Illustration of DF setup.

Decay has to offset compactification.

The GBU(F)-model

Consider a toy model for GR in GHG:

$$\Box g = 0, \quad \Box b \simeq \frac{1}{R}\partial_T f + (\partial_T g)^2, \quad \Box u \simeq \frac{2}{R}\partial_T u,$$

with free choice of the equation of motion for f.

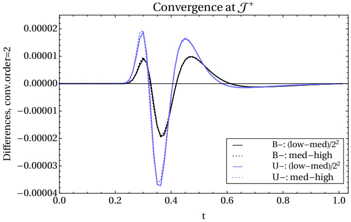
- Interesting case I: $f = 0$. Gives $b \sim \log(R)/R$
- Interesting case II: $\Box f = \frac{2}{R}\partial_T f + 2(\partial_T g)^2$. Gives $b \sim 1/R$

Analogy to GR?

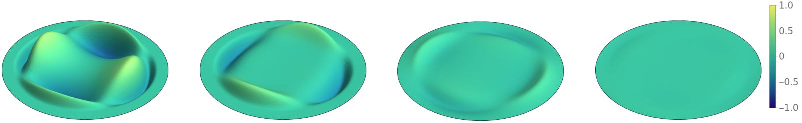
Hyperboloidal numerics for GBU(F)-model

Numerical sanity check:

- ‘Radiation fields’ evolved.
- Implemented in spherical FD code & NRPy+.
- Spectral numerics desirable too; be patient!



Snapshots of the ugly field



Spherical GR I

Can we capture stratification without extra structure? Defining suitable null vectors σ and $\underline{\sigma}$, the field equations take the form:

$$-D_\sigma D_\sigma \mathring{R} + \tfrac{1}{\kappa} D_\sigma \mathring{R}(D_\sigma C_+ - D_{\underline{\sigma}} C_+) = 4\pi \mathring{R}\, T_{\sigma\sigma}\,,$$
$$-D_{\underline{\sigma}} D_{\underline{\sigma}} \mathring{R} + \tfrac{1}{\kappa} D_{\underline{\sigma}} \mathring{R}(D_\sigma C_- - D_{\underline{\sigma}} C_-) = 4\pi \mathring{R}\, T_{\underline{\sigma\sigma}}\,,$$

plus

$$\tfrac{1}{2}\Box_2 \delta - D_a \left[\frac{e^\delta}{\kappa^2} \left(\sigma^a D_\sigma C_- - \underline{\sigma}^a D_{\underline{\sigma}} C_+ \right) \right] + \frac{2}{\mathring{R}^3} M_{\mathrm{MS}}$$
$$+ \frac{e^\delta}{\kappa^3} \left[D_{\underline{\sigma}} C_+ D_\sigma C_- - D_\sigma C_+ D_{\underline{\sigma}} C_- \right] = \frac{8\pi\, T_{\theta\theta}}{\mathring{R}^2} + 8\pi \frac{e^\delta}{\kappa} T_{\sigma\underline{\sigma}}\,,$$
$$\Box_2 \mathring{R}^2 - 2 + 16\pi \frac{e^\delta}{\kappa} \mathring{R}^2\, T_{\sigma\underline{\sigma}} = 0\,,$$

... which is *surprisingly* pretty!

Spherical GR II

Imposing GHG gives

$$D_\sigma\left(\tfrac{2}{\kappa}\mathring{R}^2 D_{\underline{\sigma}} C_+\right) + \mathring{R} D_\sigma\left(\mathring{R} F^\sigma\right) - \tfrac{1}{\kappa} D_\sigma \mathring{R}^2 D_\sigma C_+ = -8\pi \mathring{R}^2 T_{\sigma\sigma}\,,$$
$$D_{\underline{\sigma}}\left(\tfrac{2}{\kappa}\mathring{R}^2 D_\sigma C_-\right) - \mathring{R} D_{\underline{\sigma}}\left(\mathring{R} F^{\underline{\sigma}}\right) - \tfrac{1}{\kappa} D_{\underline{\sigma}} \mathring{R}^2 D_{\underline{\sigma}} C_- = 8\pi \mathring{R}^2 T_{\underline{\sigma\sigma}}\,,$$

for the speeds, and

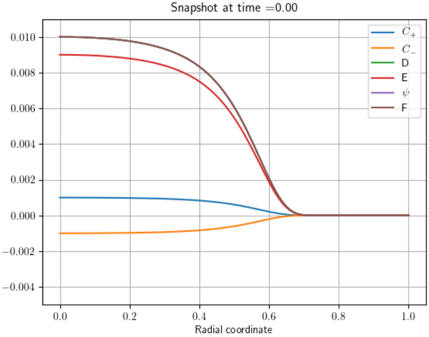
$$\Box_2\delta + D_a(\mathrm{g}^a{}_b F^b) + \frac{2e^\delta}{\kappa^3}\left[D_{\underline{\sigma}} C_+ D_\sigma C_- - D_\sigma C_+ D_{\underline{\sigma}} C_-\right]$$
$$+\frac{2}{\mathring{R}^2}\left(1 - \tfrac{2M_{\mathrm{MS}}}{\mathring{R}}\right) = \frac{16\pi\, T_{\theta\theta}}{\mathring{R}^2}\,,$$
$$\Box_2 \mathring{R}^2 - 2 = -16\pi \frac{e^\delta}{\kappa} \mathring{R}^2 T_{\sigma\underline{\sigma}}$$

for the det variables... which is *still* surprisingly pretty!

Hyperboloidal numerics for spherical GR I

Numerical work with spherical GR, basic dynamics:

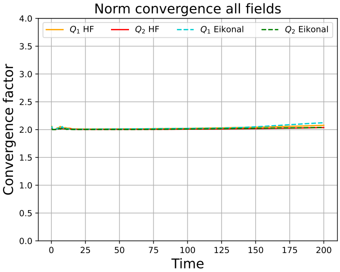
- Constraint violating ID.
- Small data disperses (reflections negligible).
- Explodes without gauge driver.



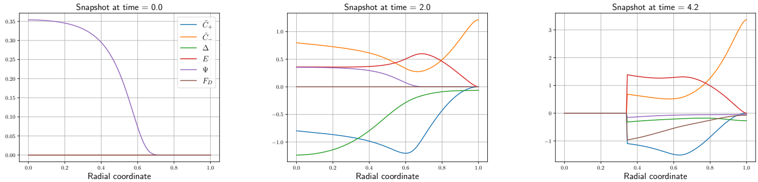
Hyperboloidal numerics for spherical GR II

Numerical work with spherical GR:

- Constraint-solved data.
- Eikonal/Height function comparable.
- Collapse, mass-loss, QNMs, tails under control.



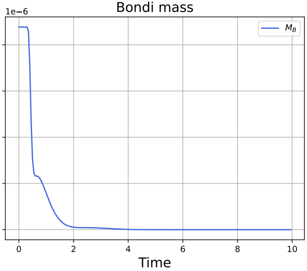
Scalar field collapse:



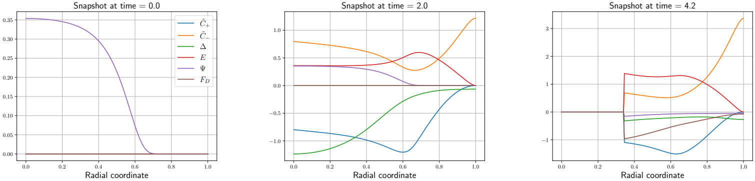
Hyperboloidal numerics for spherical GR II

Numerical work with spherical GR:

- Constraint-solved data.
- Eikonal/Height function comparable.
- Collapse, mass-loss, QNMs, tails under control.



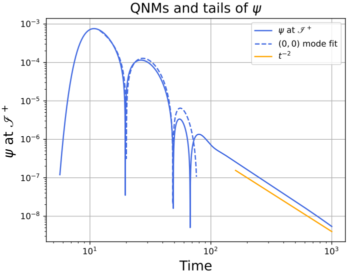
Scalar field collapse:



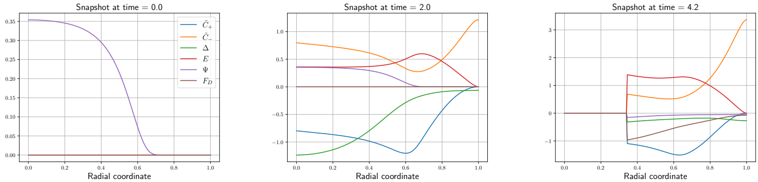
Hyperboloidal numerics for spherical GR II

Numerical work with spherical GR:

- Constraint-solved data.
- Eikonal/Height function comparable.
- Collapse, mass-loss, QNMs, tails under control.



Scalar field collapse:



Conclusions

Motivated by need for GWs at null infinity we are developing a method that employs compactified hyperboloids in NR.
Features/status:

- Dual-foliation formalism and exploitation of null-structure.
- Careful choice of gauge and constraint addition to suppress spurious radiation fields (and logs).
- Spherical GR under control.

Stay tuned for 3d GR numerics!