Conformal diagrams for strong-field hyperboloidal slices

Alex Vano-Vinuales

CENTRA, Instituto Superior Técnico

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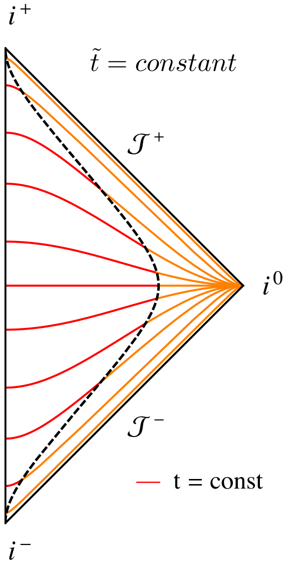
Based on 2311.04972 [gr-qc],
material in https://github.com/alexvanov/HypPenroseDiagrams.

Visualisation of spacetime slices

Conformal Carter-Penrose diagrams:

- include the whole spacetime via a coordinate compactification,
- illustrate causal properties,
- show how the chosen coordinates $(\tilde{t}, \tilde{r})$ foliate spacetime.



Visualisation of spacetime slices

Conformal Carter-Penrose diagrams:

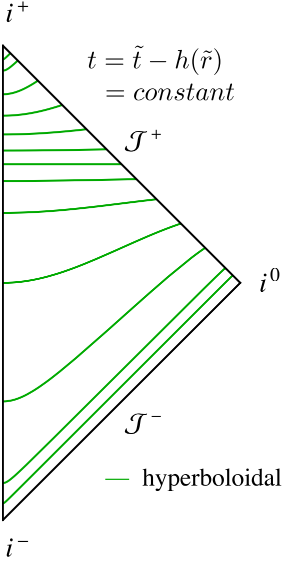
- include the whole spacetime via a coordinate compactification,
- illustrate causal properties,
- show how the chosen coordinates $(\tilde{t}, \tilde{r})$ foliate spacetime.

Change the time coordinate:

$$\tilde{t} = t + h(\tilde{r})$$

Hyperboloidal slices

- spacelike and smooth slices
- that reach $\mathscr{I}^+$.



Height function from metric

Line element in spherical symmetry:

$$
\begin{aligned}
ds^2 &= g_{tt} dt^2 + 2 g_{tr}\, dt\, dr + g_{rr}\, dr^2 + g_{\theta\theta}\, r^2\, d\sigma^2, \\
ds^2 &= -\left(\alpha^2 - \frac{\gamma_{rr}}{\chi} {\beta^r}^2\right) dt^2 + 2 \frac{\gamma_{rr}}{\chi} \beta^r dt\, dr + \frac{\gamma_{rr}}{\chi}\, dr^2 + \frac{\gamma_{\theta\theta}}{\chi}\, r^2\, d\sigma^2.
\end{aligned}
$$

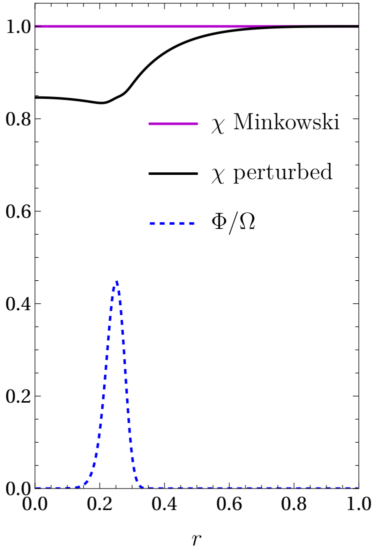
Obtain the derivative of the height function from the metric:

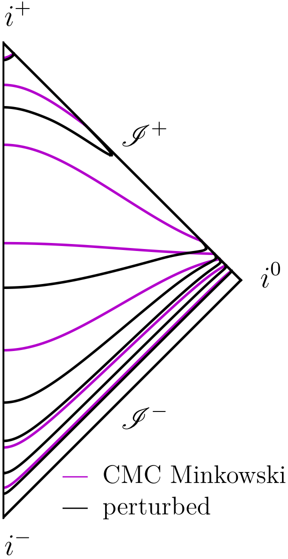
$$
h'(r) = \frac{g_{tr}}{g_{tt}} = -\frac{\frac{\gamma_{rr}}{\chi} \beta^r}{\alpha^2 - \frac{\gamma_{rr}}{\chi} {\beta^r}^2}
$$

For any (spherically symmetric) chosen metric.

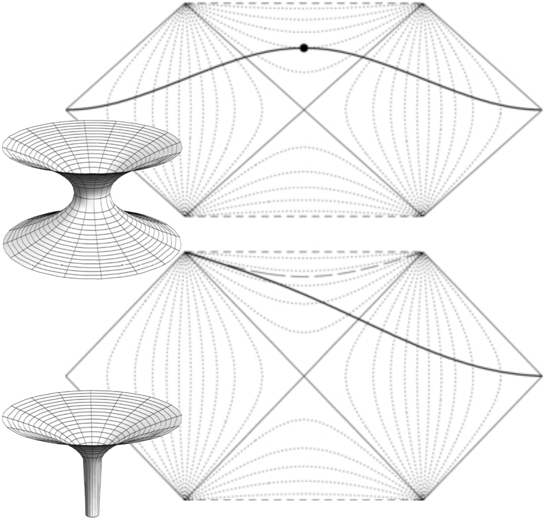
Here focus on CMC slices for closed-form expressions.

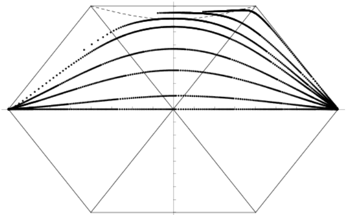
Example: perturbed regular spacetime





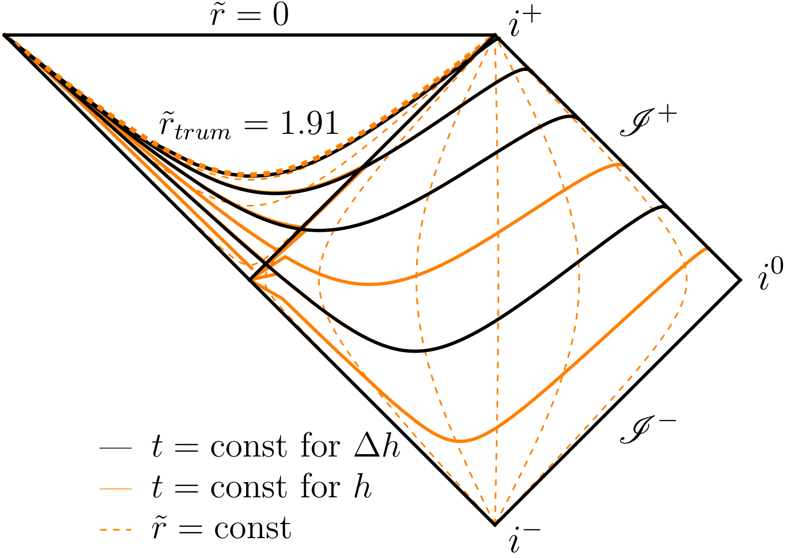
Wormhole to trumpet geometry in puncture evolution



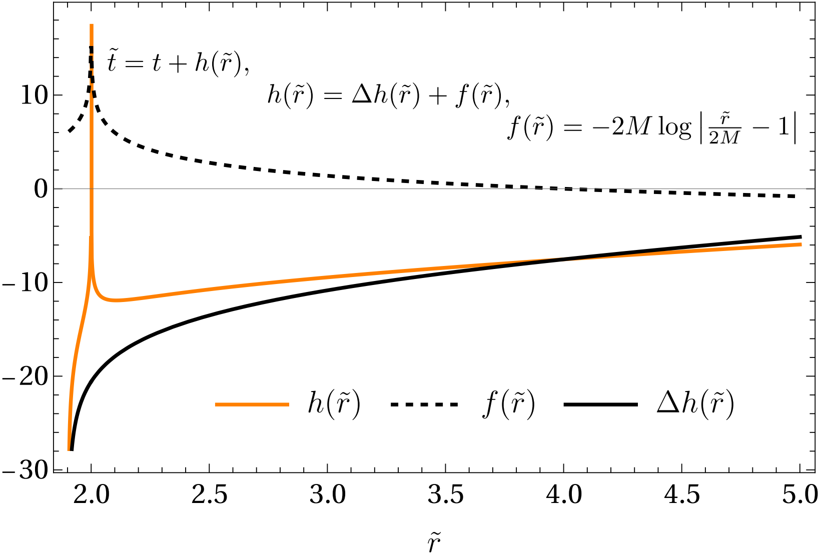


Punctures, (Baumgarte, 2011, Class. Quantum Grav. 28 215003; Hannam et al, 2008, Phys.Rev. D78 064020).

Hyperboloidal CMC Schwarzschild trumpet foliation



Height function for a trumpet slice



Height function calculation

The height function can be integrated in terms of

- the uncompactified radial coordinate $\tilde{r}$ or
- the compactified radial coordinate r.

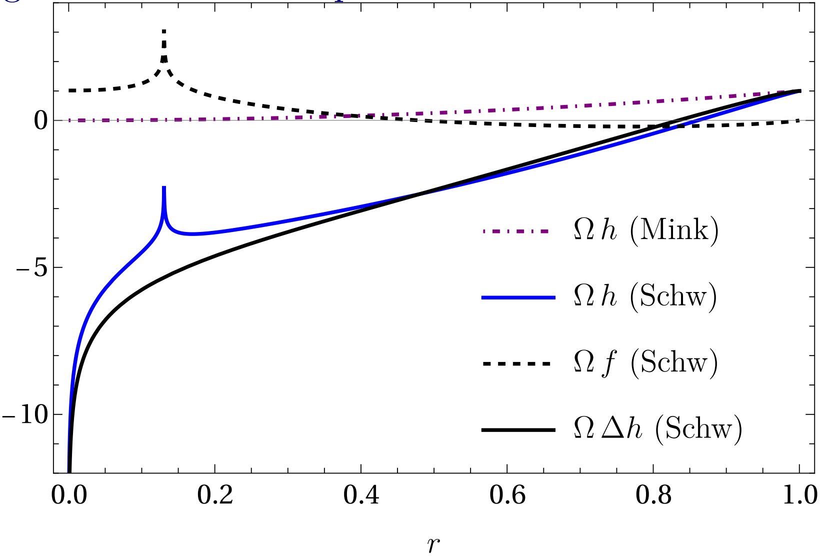
Compactification $\tilde{r} = \frac{r}{\bar{\Omega}}$, $\bar{\Omega}|_{\mathscr{I}} = 0$ and $r|_{\mathscr{I}} = r_{\mathscr{I}}$.

Conformal rescaling $g_{\mu\nu} = \Omega^2 \tilde{g}_{\mu\nu}$, $\Omega|_{\mathscr{I}} \sim \bar{\Omega}|_{\mathscr{I}}$.

For closed-form or numerical compactified data ($c \neq$ light speed):

$$\begin{aligned}\Delta h'(r) &= h'(r) - f'(r) = \frac{g_{tr}/c}{g_{tt}/c^2} - \frac{\Omega^2 + g_{tt}/c^2}{g_{tt}/c^2}\left(\frac{\bar{\Omega} - r\bar{\Omega}'}{\bar{\Omega}^2}\right) \\ &= \frac{\left[\Omega^2 c^2 - \left(\alpha^2 - \frac{\gamma_{rr}}{\chi}{\beta^r}^2\right)\right]\left(\frac{\bar{\Omega} - r\bar{\Omega}'}{\bar{\Omega}^2}\right) - \frac{\gamma_{rr}}{\chi}\beta^r c}{\alpha^2 - \frac{\gamma_{rr}}{\chi}{\beta^r}^2}.\end{aligned}$$

Height function on compactified slice



Evolve the Penrose diagram quantities

- Ingoing and outgoing null coordinates, $\tilde{u}$ and $\tilde{v}$, satisfy

$$\partial_{\tilde{t}}\tilde{u} = -\tilde{c}_+\partial_{\tilde{r}}\tilde{u}\,, \quad \partial_{\tilde{t}}\tilde{v} = -\tilde{c}_-\partial_{\tilde{r}}\tilde{v} \quad \text{with} \quad \tilde{c}_\pm = \left(\pm\tilde{\alpha}\sqrt{\frac{\tilde{\chi}}{\gamma_{\tilde{r}\tilde{r}}}} - \beta^{\tilde{r}}\right),$$

from the Eikonal equations, $\tilde{g}^{\tilde{u}\tilde{u}} = \tilde{g}^{ab}\nabla_a\tilde{u}\nabla_b\tilde{u} = 0 = \tilde{g}^{\tilde{v}\tilde{v}}$.

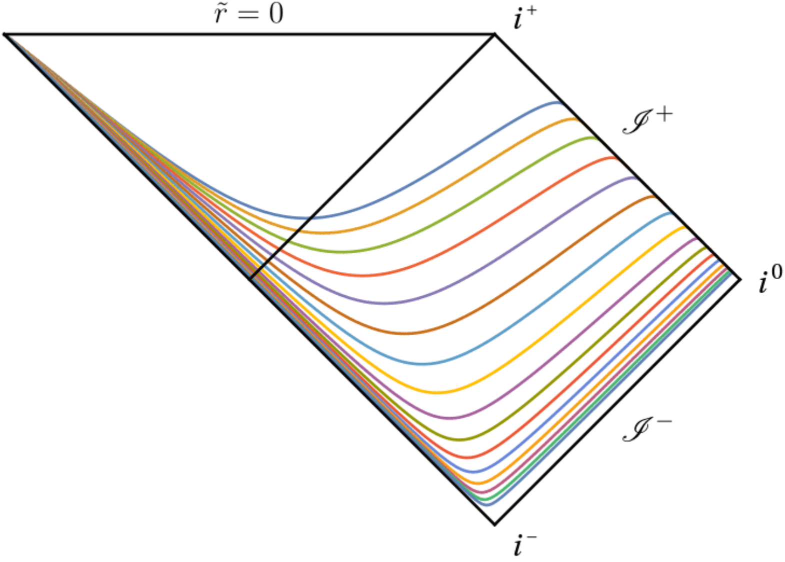
- Evolve instead hyperboloidal compactified initial data in

$$R = \frac{1}{2}(V-U)\,, \quad T = \frac{1}{2}(U+V)$$

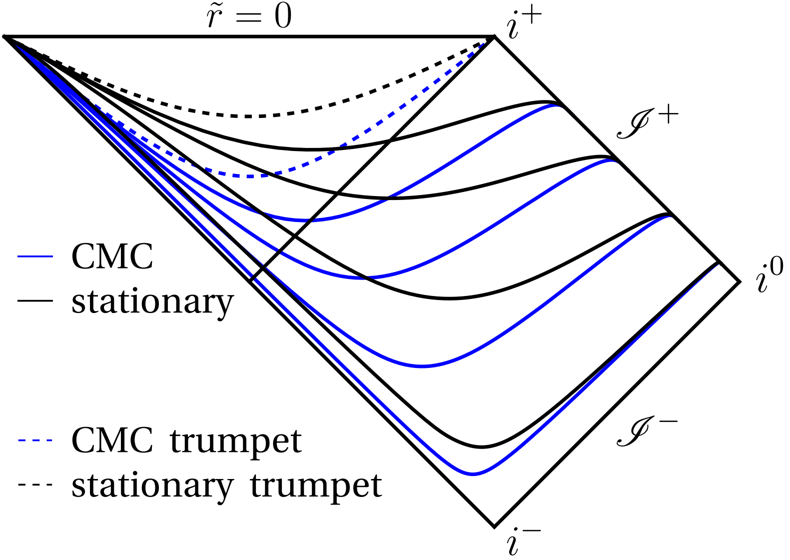
with $\quad \partial_t R = \beta^r\partial_r R + \alpha\sqrt{\frac{\chi}{\gamma_{rr}}}\partial_r T\,, \quad \partial_t T = \beta^r\partial_r T + \alpha\sqrt{\frac{\chi}{\gamma_{rr}}}\partial_r R\,.$

- Plot T as a function of R in a Penrose diagram.

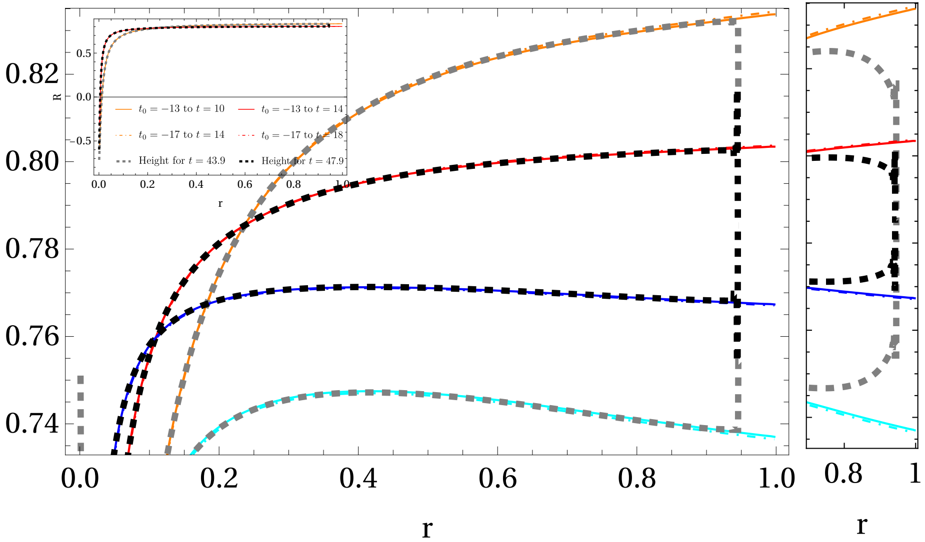
Evolution of hyperboloidal trumpet slices



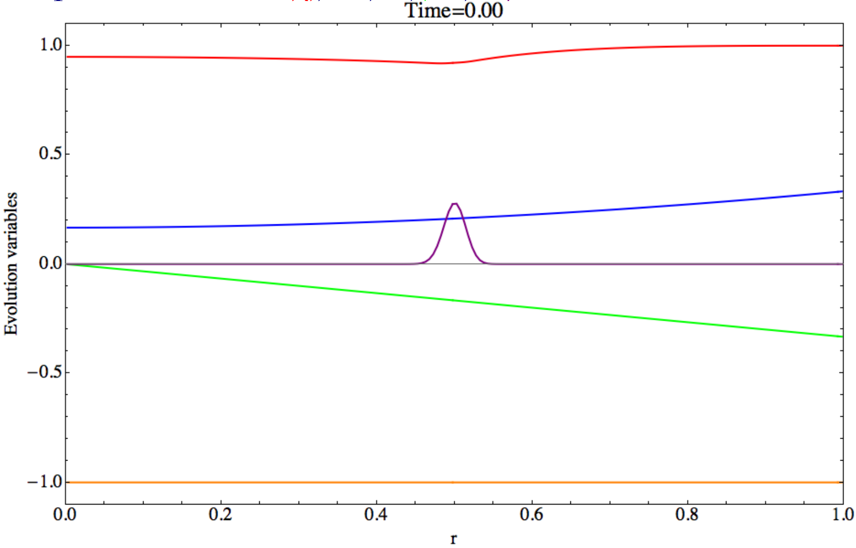
Comparison between CMC and relaxed slices



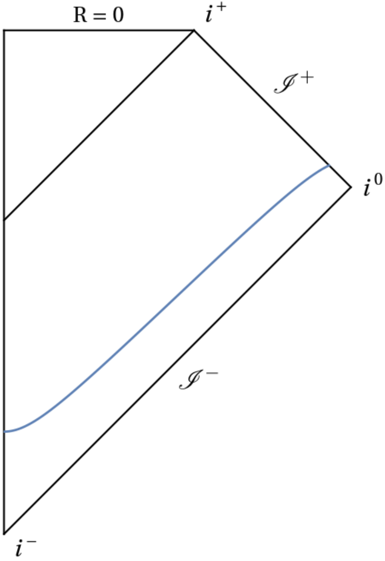
Comparison: height function to Eikonal



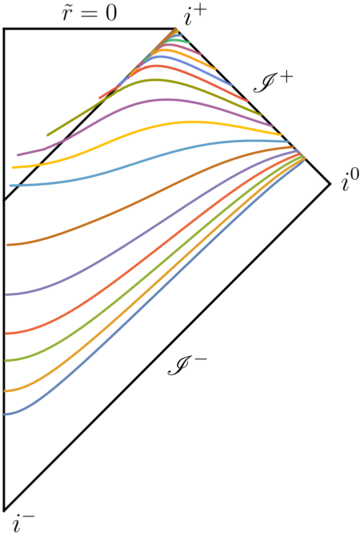
Collapse evolution: $\chi, \tilde{K}, \alpha, \beta^r, \Phi/\Omega$

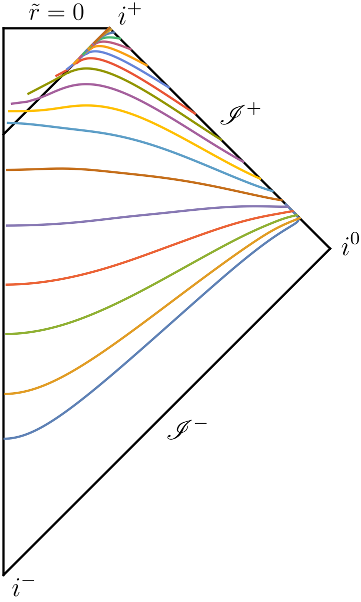


Evolution of collapse hyperboloidal slices



Different initial slices



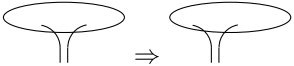


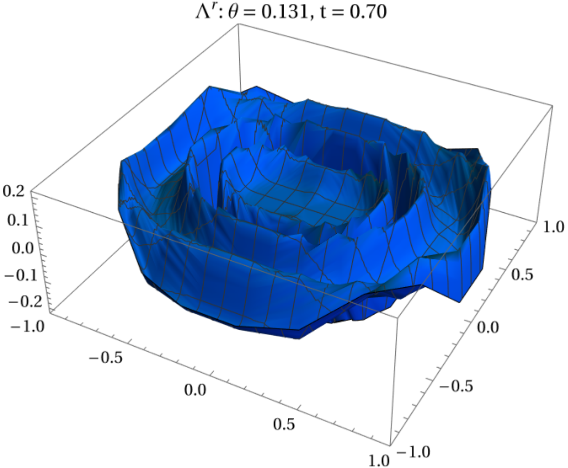
Related projects

Black hole initial data in trumpet form for hyperboloidal punctures

– going beyond spherical symmetry makes things more complicated.

Hyperboloidal Bowen-York initial data for boosted and spinning binaries by Buchman, Pfeiffer and Bardeen. *Phys.Rev.* D80 (2009).

- Kerr trumpet slices – compare to (asymptotically) CMC slices Schinkel, Panosso Macedo and Ansorg. *Class. Quant. Grav.* 31 (2014), Schinkel, Ansorg and Panosso Macedo. *Class.Quant.Grav.* 31 (2014).
- Off-centered (Schwarzschild) trumpet 
- Boosted trumpet
- Two trumpets (head-on collision, binary)



Thanks for listening! Questions?